Similarity Solutions for Stellar Structures

G A Wynn

Consider a family of chemically homogeneous stars which have different masses M and radii R, but are similar in the sense that the density $\rho(r)$ and pressure $P(r)$ at radius r can be written as universal functions of the reduced radius $x = r/R$. Let these functions be given as

$$\rho(r) = \frac{M}{R^3} F_\rho(x)$$

$$m(r) = MF_m(x)$$

where $m(r)$ is the mass within a radius r and $F_m(1) = 1$, $F_\rho(1) = 0$. To illustrate the method we consider a polytrope of index 3 with perfect gas equation of state, so $P = K\rho^3$

$$P = \frac{\rho}{\mu m_H} kT$$

Substituting in the equation of hydrostatic equilibrium

$$\frac{dP}{dr} = \frac{Gm\rho}{r^2}$$

we get

$$\frac{1}{R}\frac{dP}{dx} = G\frac{MF_m(x)}{R^3}\frac{MF_\rho(x)}{R^3}\frac{R^2}{x^2}$$

so

$$\frac{dP}{dx} = \frac{M^2}{R^4}\left(G\frac{F_m(x)F_\rho(x)}{x^2}\right)$$

from which we deduce

$$P(r) = \frac{M^2}{R^4} F_P(x)$$

From the equation of state it follows that

$$T(r) = \frac{M}{R} F_T(x)$$

and from the polytropic equation we obtain the mass-radius relation

$$M \propto R^2$$

by putting, for example $x = 0$.

It should be possible to extend the method to models of stars in radiative equilibrium. In particular, for stars approaching the main sequence, we should be able to derive the luminosity and power generated by nuclear reactions as functions of M and R.