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A2 6 Heads or Tails

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Abstract

This paper applies initial conditions to a coin by using classical mechanics to determine the outcome of a coin flip. If a 2p coin is flipped 0.5 meters with a total energy of 0.17 J, by finding the angular momentum of the coin the uncertainty disappears and the coin is found to land opposite side up to the initial setup.

Introduction

The probability for each outcome when flipping a coin is agreed to be 50:50, however like most things in nature, randomness and probability is a human convention assigned to systems where the true nature is unknown or too complex. A coin's outcome is only unknown until the initial conditions of a coin are determined, allowing a precise outcome of the coin to be predicted using classical mechanics. In this paper initial conditions will be applied to a 2p coin to determine the outcome of the coin with certainty.

Method

To model a convectional coin flip, a coin of mass m and radius R is flipped into the air at height h. During the flip, the coin will receive mechanical energy to achieve a height of h as well as rotational energy which allows the coin to rotate during the flip. The total energy E_t of the coin in joules is therefore:

$$E_t = mv_y^2/2 + mgh + I\omega^2/2 \qquad (1)$$

With terms vertical kinetic energy, gravitational potential energy and rotational energy respectively. I is the moment of inertia (kg m^2) and ω is the angular velocity of the coin's rotation (rad s^{-1}). The angular velocity is required to find whether the outcome is heads or tails when the coin falls, where one of these outcomes will occur based on the angular velocity of the coin. The kinetic and potential energies will oscillate at different points of the flip, however the sum of these two energies is a constant which can be calculated at the maximum height of h metres when there is no kinetic energy. Similarly, kinetic energy will be a maximum the moment the coin is flipped, so from the sum of the mechanical energy, the initial velocity u_y of the coin as it is tossed can determine the total time t the coin travels as it is tossed and falls back down using classical mechanics.

After finding the mechanical energy, the rotational energy can be calculated if the total energy is known. The moment of inertia of a coin which is assumed to be a disc of mass m and radius R is found using [1]:

$$I = MR^2/4 \qquad (2)$$

Using this result, the angular velocity of the system can be found by rearranging (1):

$$\omega = (2(E_t - (E_k + E_p))/I)^{0.5} \qquad (3)$$

With the angular velocity determined, the coin's outcome can be found using the final angle between the coin and an arbitrary axis of the coin's initial position after time t. This is done by finding how many full rotations the coin experiences with angular velocity ω as the full angle the coin rotates is simply ωt . If the coin is initially head side up, then the arbitrary angle and axis chosen is:

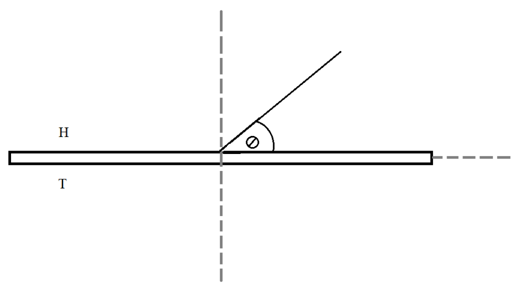

Figure 1: Axis relative to coin's initial setup

Where the rotation will occur counter-clockwise relative to this setup. From figure 1, if the final angle from the axis is $\pi/2 > \theta > 3\pi/2$, then is heads, while if the angle is $\pi/2 < \theta < 3\pi/2$, then the outcome will be tails - assuming there is no bouncing when the coin lands, and that it lands face down on the side closest to the surface it lands on. To determine the full angle the coin rotates, the time taken for the coin's flip can be calculated using motion mechanics. Using the displacement equation:

$$0 = u_y t + gt^2/2 \tag{4}$$

Where u_y is the initial velocity which is calculated from the kinetic energy from when the coin is initially flipped which is simply the total mechanical energy. The time taken for the coin to return to $s = 0$ (back to the hand) can be found by solving the quadratic. When the final angle is found, it will be divided by 2π (a full rotation) to find the total number of rotations. From there, the first decimal number after the units will decipher the orientation of the coin as it lands, and thus whether it is heads or tails.

Results

Assuming an elastic system and no air resistance, choosing a 2p coin of $m = 7.12 \times 10^{-3}$ kg and $R = 2.59 \times 10^{-2}$ m [2], the moment of inertia is found to be $I = 2.985 \times 10^{-7}$ kg m^2. Using the total energy of the average flick to be $E_t = 0.17$ J [3], the maximum potential energy achieved at a height of 0.5 m (reasonably chosen arbitrary value) is $E_k + E_p = 0.0349$ J. From this the angular velocity can be found using equation 3, where $\omega = 16.6$ rads^{-1}.

Solving equation 4 using $u_y = 3.1321$ m s^{-1} which is found from $E_k = 0.0349$ J, the time of the coin's flip from toss to fall is $t = 0.639$ s. So by multiplying this result by ω the total angle the coin flipped is 10.61 radians, which when divided by 2π gives 1.68 full rotations. The 1.00 full rotation is irrelevant, but the 0.68 rotations is the desired value needed to determine the outcome, and so $0.68 \times 2\pi = 4.32$ radians. From figure 1, assuming the coin is initially heads, the coin will be flipped to tails as:

$$\pi/2 < 4.32 < 3\pi/2. \tag{5}$$

So if an initial energy of 0.17 J is applied to flip a coin, it will always result in the opposite outcome to the initial coin orientation, given the initial conditions. This is a classical analogue to quantum uncertainty, where by finding the nature of a system (operating), the uncertainty disappears. By not always knowing values like the height a coin travels or the total energy of a coin, the probability of a coin's outcome is 50:50 and therefore not always certain.

Bibliography

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[3] *Quora. (n.d.). How much energy is in the average flick? [Last Accesssed 05 Dec 2023*