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A3 4 In A Hurry?

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Abstract

In this paper, we investigated the speed one would have to travel at for a red traffic light to appear green due to the Doppler effect. We determined that if a vehicle was travelling at a speed of 9.7×10^7 m s^{-1} (or 30% the speed of light), a red traffic light would appear green due to the Doppler Effect, allowing you to continue on your high-speed journey uninterrupted.

Introduction

Have you ever been driving to an appointment, running late, and then a traffic light turns red, forcing you to stop whilst you gradually get more and more irritated the longer you wait? Well, due to the Doppler Effect, if you were travelling fast enough that red light may look green instead, which is a legally dubious method of allowing you to continue on your journey. As a result, we investigated at what speed this extraordinary scenario may be possible.

The Doppler Effect

The Doppler Effect occurs when a moving object emits a wave (typically sound or light). A common example is that of an emergency vehicle driving past a pedestrian with its siren blaring. When the vehicle passes the pedestrian, the pitch of the siren audibly changes, even though the siren is constantly emitting the same frequency. This is because, when the vehicle is approaching the pedestrian, the sound wave peaks are compressed as they are being emitted in the direction of the vehicle's motion, resulting in a higher frequency (or higher pitch) and, therefore, a shorter wavelength. When the vehicle moves past the pedestrian, the wave peaks become more spread out, resulting in a lower frequency and a longer wavelength. This is clearly illustrated in Figure 1.

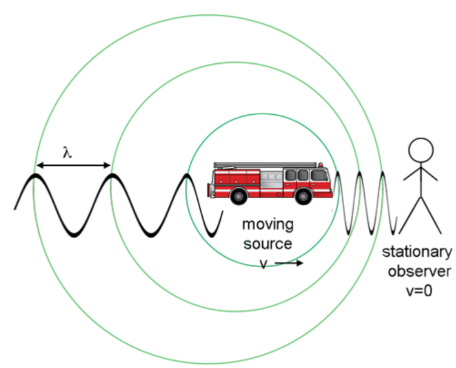

Figure 1: Illustration of how the Doppler Effect alters the wavelength of a moving source [1]

In the case of light, an excellent example is the red-shift and blue-shift seen in distant stars. Stars moving away from us look as if their distance between each wave peak increases, resulting in a decrease of the frequency. This makes their light appear redder - hence the term red-shift. The opposite is true for blue-shift; the star is moving towards us so its frequency increases, resulting in bluer light. This can be seen in Figure 2.

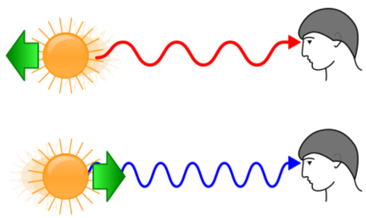

Figure 2: Illustration of how the Doppler Effect alters the wavelength and, therefore, the colour of light emitted by a moving source [2]

From Red to Green

Applying this knowledge of the Doppler Effect, if we were to travel towards a stationary traffic light at a high enough speed, we may be able to blue-shift the red stop light until it appears more like a green light. To determine this speed, we began with the relativistic version of the Doppler formula, as length contraction effects become relevant. The relationship between the observed wavelength and the wavelength from the source is given below [3]:

$$\lambda_{obs} = \lambda_s \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} \tag{1}$$

In Equation 1, λ_{obs} is the observed wavelength (green), λ_s is the source wavelength (red), v is the velocity of our car and c is the speed of light in a vacuum. Rearranging Equation 1 to make the velocity the subject of the equation gives:

$$v = \frac{\lambda_s^2 - \lambda_{obs}^2}{\frac{\lambda_s^2}{c} + \frac{\lambda_{obs}^2}{c}} \tag{2}$$

Green light has a wavelength ~500 nm, whereas red has a wavelength ~700 nm. As a result, for our calculations: $\lambda_{obs} = 500$ nm and $\lambda_s = 700$ nm. Inputting these values in gives a velocity, v, of 9.7×10^7 m s^{-1}, which can also be expressed as 2.2×10^8 mph or around 30% the speed of light ($0.3c$). At this speed, you could travel around the circumference of the Earth in less than half a second [4]!

In order to find what colour a traffic light would have to emit in order to appear red to our relativistic driver, we rearranged Equation 1 to get:

$$\lambda_s = \lambda_{obs} \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{-0.5} \tag{3}$$

Inputting in $v = 0.3c$ and $\lambda_{obs} = 700$ nm (red light), we get $\lambda_s \approx 950$ nm. This would be in the infrared range, normally beyond human vision but now visible due to the Doppler Effect!

Conclusion

We have determined that the speed a car would have to travel for a red traffic light to appear green would be around 30% the speed of light. This incredible (and unrealistic) speed is fast enough to ensure that no traffic lights hinder your journey (and is fast enough to travel anywhere in the world almost instantaneously). However, it would appear from a police officer's reference frame that you ignored a red light whilst speeding, so you could be liable to being charged with a number of traffic violations.

References

[1] NASA, Imagine The Universe!, `https://imagine.gsfc.nasa.gov/features/yba/M31_velocity/spectrum/doppler_more.html` [Accessed 27 November 2023]

[2] Kidzsearch Wiki, Blue Shift, `https://wiki.kidzsearch.com/wiki/Blue_shift` [Accessed 27 November 2023]

[3] Libre Texts, Doppler Effect for Light, `https://rebrand.ly/ns4wukn` [Accessed 27 November 2023]

[4] Cool Cosmos, How large is Earth?, `https://coolcosmos.ipac.caltech.edu/ask/57-How-large-is-Earth-` [Accessed 27 November 2023]