Journal of Physics Special Topics
An undergraduate physics journal

P4_7 Santa's Jet

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December 12, 2022

Abstract

In the movie Elf Santa replaces the waning Christmas spirit with a jet engine. We have calculated the speed required to allow Santa's sleigh filled with presents to fly. This speed is $5500(\pm1700)\text{ms}^{-1}$. This would require a thrust produced by the engine of $3.8(\pm2.2) \times 10^7$N.

Introduction

In the movie Elf, Santa replaces the waning Christmas spirit with a jet engine. Ref.[1] describes the results of a study on the aerodynamics of Santa's sleigh it found that the traditional sleigh would result in a lift coefficient of "-0.06". This would result in 425N of force acting towards the Earth's surface. As a result, this paper will assume Santa also decided to add two wings from a Boeing 747 aircraft in order to create the necessary amount of lift. In this paper, we will attempt to quantify the speed required for Santa to remain in level flight and estimate the thrust produced by the jet engine to equal the drag at this speed.

Weight of Santa's Sleigh

The weight of Santa's Sleigh must be calculated, in this paper we will consider the mass of the sleigh and the mass of the presents only. The mass of the reindeer and Santa have been excluded as this will be negligible. Ref.[2] suggests that the mass of a British naval sled, used from 1875-1876, is 635kg. This does not represent modern sleigh technologies, however as there are approximately 1.98 billion children on Earth the mass of the sleigh will have little impact on the values calculated later on. (Value used from Ref. [3]) Santa only delivers presents to children who have been good this year, therefore a Niceness rate must be taken into account, in this paper we will estimate this to be 0.5. This is done by modelling the naughty or nice list, as there are only two possible outcomes a simple estimate can be achieved by assuming an even split between naughty and nice. We will allow an uncertainty in this rate of ±0.3. Eq.(1) is the summation of the weight of the sleigh and the weight of the presents.

$$W = Ncmg + M_s g \tag{1}$$

Where W is the combined weight, N is the Niceness rate, c is the number of children, m is the average mass of a present assumed to be 0.5kg, g is 9.81ms^{-2}, M_s is the mass of the sleigh. Using this equation leads to a weight of $4.85(\pm2.91) \times 10^9$N. In the next section the weight of the sleigh will be used to calculate the speed required to maintain a level flight.

Speed of Santa's Sleigh

The speed of Santa's sleigh will be calculated by equating the weight of Santa's sleigh with the lift on Santa's sleigh. The upthrust that keeps planes and sleighs from plummeting out of the sky is generated by the difference in the speed of air flow above and below the wing. The top part of the wing experiences air moving quicker than the bottom section of the wing, therefore an imbalance in pressure is set up causing the upthrust on the aircraft. The equation describing the upthrust experienced by a sleigh is described below.

$$v = \sqrt{\frac{U}{0.5\rho S C_L}} \qquad (2)$$

Where U is the upthrust, ρ is the density of air, v is the speed of Santa's sleigh, S is the surface area of the wing, C_L is the coefficient of lift. This section relies heavily on the assumption that Santa fitted a pair of wings on his sleigh. This is a requirement as the traditional shape of Santa's sleigh would provide a down force rather than an upthrust. In this paper we assume he fits two wings from a Boeing 747, according to Ref.[4] the coefficient of lift would be 0.52 and would have a wing area of 510.97m^2 and the density of air used in this paper is 1.2kgm^{-3}. By equating the upthrust with the weight of Santa's sleigh, the velocity of Santa's sleigh can be determined. This was found to be $5500(\pm1700)\text{ms}^{-1}$. This is approximately 10 times the speed of sound. Future papers could look at the effect the reduction in the density of air has on the speed required to keep Santa's sleigh flying. The next section will attempt to quantify the thrust required to keep the velocity constant and compare that to the number of jet engines required to cause such a thrust.

Thrust of Jet Engines

In order for Santa's sleigh to remain in the air, the thrust must balance out the drag forces experienced by the sleigh. The drag forces can be found by using Eq.3. Santa's sleigh has been approximated as a rectangular cross sectional area with the height being a person sitting down found in Ref.[5] to be 0.910m and the width being a person laying down, found in Ref.[6] to be 1.74m.

$$T = 0.5 A C_D v^2 \rho \qquad (3)$$

Where A is the cross-sectional area, $C_D = 1.32$ is the drag coefficient, ρ is the density of air, v is the speed of Santa's sleigh. This leads to a value of thrust to be $3.8(\pm2.2) \times 10^7\text{N}$. This is a lot of thrust therefore Santa must have access to a brand new type of jet engine technology in order to keep his sleigh flying.

Conclusion

In this paper we have estimated that in order for Santa to replace Christmas spirit with a jet engine. He would have to produce a speed of his sleigh of $5500(\pm1700)\text{ms}^{-1}$. This would require the engine to produce a thrust of $3.8(\pm2.2) \times 10^7\text{N}$, therefore Santa must have had access to a new type of jet engine technology capable of replacing the effects of Christmas spirit.

References

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