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A3_9 All I Want Thaw Christmas Is You

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Abstract

In this paper, we investigate if it would be possible to defrost Mariah Carey from an ice cube of length 2 m in time for the 1st of December if you began defrosting her on the 1st of November. It was found that it would take a total of 12.6 days for Mariah to return to normal body temperature, leaving her plenty of time to prepare for the Christmas season before it arrives.

Introduction

The Christmas season is upon us again. At this time, the world bears witness to seasonal magic that only comes about once a year. A main figurehead in this magic is the Queen of Christmas herself, Mariah Carey. It is often joked that Mariah is frozen between the Christmas seasons and allowed to defrost every year in time for the celebration to begin. In this paper, we investigated how long it would take for Mariah to defrost if she were encased in an ice cube of length 2 m.

Model and Assumptions

For the sake of simplicity, we modelled Mariah as a cylinder with height 1.73 m [1] and diameter 36.7 cm (the average shoulder width for an American woman) [2]. We built our model on the assumption that Mariah is defrosting in a room with an ambient temperature $T_a = 40$ °C $= 313$ K, as this is above the average body temperature of $T = 37$ °C $= 310$ K [3] that Mariah must reach. We also assumed that the room is continually heated such that it never drops below T_a. Furthermore, we assume the ice is at a temperature of $T_i = 0$ °C $= 273$ K. Finally, we chose to calculate the time for the ice cube to melt and the time for Mariah's body temperature to reach T separately. In reality, Mariah's body would begin to acclimate unevenly as soon as any part of her body were freed from the ice, but the maths of this would be unnecessarily complicated for a paper of this scope.

Melting the Ice Cube

We began by calculating how long it would take for the ice to melt. To do this, we applied Fourier's law of thermal conduction:

$$\frac{Q}{t_i} = kA_i\frac{\Delta T_i}{\Delta x}. \quad (1)$$

Here, Q is the heat transferred to the ice, t_i is the time taken for the ice to melt, k is the ice's thermal conductivity (2.75 W m^{-1} K^{-1} [4]), A_i is the surface area of the ice (24 m^2) and $\frac{\Delta T_i}{\Delta x}$ is the ice's temperature gradient (the difference in the ice's temperature at its surface and centre, 40 K, over the distance from the surface to the centre, 1 m; 40 K m^{-1}). We can substitute $Q = m_iL_f$, where m_i is the ice's mass and L_f is the ice's latent heat of fusion (334×10^3 J kg^{-1}). Likewise, we can substitute $m_i = V_i\rho_i$, where V_i is the ice's volume and ρ_i is the ice's mass density (917 kg m^{-3}). Here, $V_i = V_{cube} - V_M$, where V_{cube} is the volume of the cube (8 m^3) and V_M is Mariah's cylindrical volume (0.183 m^3). This gives $m_i = 7170$ kg. Rearranging for t_i:

$$t_i = \frac{m_i L_f}{k A_i} \left(\frac{\Delta T_i}{\Delta x} \right)^{-1}. \quad (2)$$

This gives $t_i = 9.07 \times 10^5$ seconds = 10.5 days.

Defrosting Mariah

Next, we calculated how long it would take for Mariah's body temperature to rise from T_i to T. This required the application of lumped system analysis [5]:

$$\frac{T - T_a}{T_i - T_a} = \exp\left(\frac{-h_c A_M t_M}{\rho_M V_M c_M} \right). \quad (3)$$

Here, h_c is Mariah's heat transfer coefficient (4.1 W m^{-2} K^{-1} [6]), A_M is Mariah's cylindrical surface area (2.21 m^2), t_M is the time taken for Mariah's body temperature to reach T, ρ_M is Mariah's mass density (which we approximate as the mass density of water, 1000 kg m^{-3}, since the human body is mostly water) and c_M is Mariah's specific heat capacity (3.47×10^3 J kg^{-1} K^{-1} [7]). Rearranging for t_M:

$$t_M = \frac{-\rho_M V_M c_M}{h_c A_M} \ln\left(\frac{T - T_a}{T_i - T_a} \right). \quad (4)$$

This gives $t_M = 1.81 \times 10^5$ seconds = 2.10 days.

Conclusion

To find the total time to defrost Mariah, we simply calculated the sum of t_i and t_M, which gives 12.6 days. This means that if you began defrosting Mariah after Halloween, on the 1st of November, she would defrost in plenty of time before the beginning of December. However, this conclusion is based on a very simplified model that does not account for Mariah's body acclimating as the ice melts, which would make the model much more realistic. Of course, this is based on a wholly unrealistic situation to begin with, as hypothermia occurs at core temperatures below 21 °C [3], but we can choose to believe that Mariah would survive through the magic of Christmas.

References

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