Journal of Physics Special Topics
An undergraduate physics journal

A3_8 Lunar Magnetic Fields?

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December 7, 2022

Abstract

In this paper, we explore the effect that a lunar magnetic field would have on the magnitude of the Earth's magnetic field at the surface. This was achieved by performing coordinate transformations to give a resultant magnetic field strength at the Earth, which was then used to create plots of the resultant magnetic field strength with altered values for radius and orbital radius of the Moon. From these, we have deduced that, for a lunar magnetic field to have a significant effect on the surface of the Earth, its orbital radius would need to be decreased by a factor of 10^4. Alternatively, the Moon's radius would have to be increased by a factor of a million.

Introduction

For many centuries, mankind has relied on compasses for navigation. However, for much of the compass' history, the underlying physics was not understood, and it was not until 1600 that William Gilbert realized that the Earth itself was a giant magnet responsible for deflecting compass needles [1]. Whilst it is known with confidence that the Moon has no magnetic field (and no evidence has been found to show that it ever did), navigation using a compass may have been different if it had. Thus we explore the effects that a Lunar magnetic field would have at the Earth's surface.

Magnetic Field Formulae

Before we begin deriving formulae for magnetic field strength, we will make the assumption that the shape of the field for the Earth and Moon is a dipole. We are also choosing to ignore additional factors such as the presence of plasma; the maths would become unnecessarily complicated, and thus a simple dipole field is a sufficient representation for the structure of the magnetic field. The equation shown below is the formula for a magnetic dipole of a spherical body [1]:

$$\vec{B}(r,\theta,\phi) = B_{eq}\left(-2\frac{R}{r}^3\cos\theta, -\frac{R}{r}^3\sin\theta, 0\right)$$

Here, B_{eq} is the surface equatorial magnetic field strength and R is the body's radius. r, θ, and ϕ hold their usual meaning for spherical polar coordinates. For completeness, we choose to consider the interplanetary magnetic field (IMF) in our calculations, although its effect on the magnetic field strength at the surface of the Earth is almost negligible [1]:

$$\vec{B_\odot}(r,\theta,\phi) = B_{r0}\left(\frac{R_\odot}{r}^2, 0, \frac{-\omega_\odot R_\odot}{V}\sin\theta\left(\frac{R_\odot}{r}\right)\right)$$

B_{r0} is the radial component at the surface of the Sun, $R_\odot$ is the solar radius.

Before we can begin combining these formulae to find the resultant field strength at the surface of the Earth, we need to consider the fact that the formula for the IMF and the Earth and Moon's dipole field each have their origin at a different location. To remedy this, we will need to make coordinate transformations for r, θ, and ϕ. Let us consider a common point, P, in both the Earth and Moon's coordinate systems. In the Moon's coordinate system, we may represent this as the vector $\vec{P}(\rho, \alpha, \beta)$ (in spherical polar coordinates). With the aid of Figure (1), and various trigonometry rules, the coordinate transformations to $\vec{P}(r, \theta, \phi)$ are listed in equations (1) and (2) (the formula for β has been omitted since it isn't used).

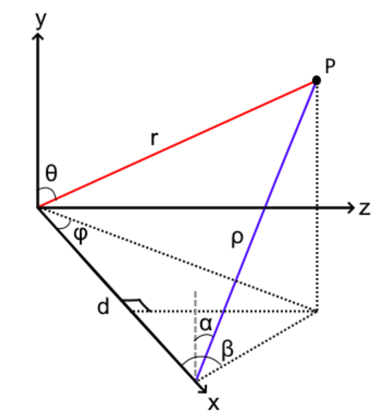

Figure 1: Diagram illustrating the geometry of the coordinate change. The red and blue lines denote the position vectors of coordinate P in the two coordinate systems.

$$\rho^2 = d^2 + r^2 - 2rd \sin\theta \cos\phi \tag{1}$$

$$\cos\alpha = \frac{r}{\rho} \cos\theta \tag{2}$$

We decide to place the Moon directly behind the Earth in the ecliptic plane, and so the distances $d = d_{\odot}$ for the Sun and $d = -d_{Moon}$ for the Moon.

Magnetic Field Plots

Finally, we can combine our formulae for magnetic field strength with the coordinate transformations applied. Through some initial calculations, it can be shown that the Moon (with equatorial magnetic field strength of 1 nT) has almost no effect on the Earth's surface field strength.

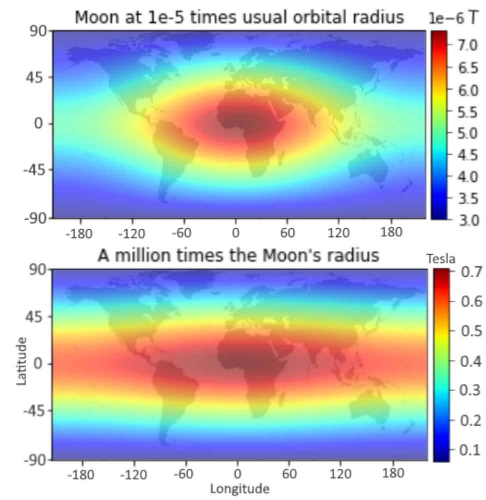

Figure 2: Plots of deviation from Earth and solar magnetic field strength on Earth's surface.

Thus, we alter d_{Moon} and R_{Moon} until a significant change is observed, which is what is shown in Figure (2). The top plot shows the deviation from the Earth-Sun configuration due to a lunar magnetic field when the distance to the Moon decreased, and the bottom when radius is increased. Altered values of d_{Moon} and R_{Moon} were chosen by changing their order of magnitude until the geometry of the field at the surface was altered.

Discussion and Conclusion

By examining the plots in Figure (2), we have found that for a change to be felt on Earth, either the distance to the Moon must have decreased by a factor of 10^4 or the Moon's radius must have increased by a factor of 10^6. These differences likely would've had an impact on compass needles, however the changes required for them to be felt are incredibly unrealistic. Furthermore, we can see that the greatest deviation from the Earth-Sun configuration occurs at a point on the equator; the point directly below where the Moon is placed.

References

[1] Margaret G. Kivelson and Christopher T. Russell. *Introduction to Space Physics*. Cambridge University Press, 1995.