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P3_9 Modelling Spider-Man's Swing as an Elastic Pendulum

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Abstract

We investigate an elastic pendulum to see if it could be used to model the trajectory of a particular Spider-Man swing. Our aim is not to find exact parameters, but rather to see if such a swing could be modelled in this way. We find that for reasonable constants (e.g. $k \sim 30\ \mathrm{Nm^{-1}}$) and initial velocities $\dot{r} = 5\ \mathrm{ms^{-1}}$ and $\dot{\theta} = 1\ \mathrm{rads^{-1}}$, it might be a useful way to describe this swing.

Introduction

In this paper we investigate a particular swing towards the end of the film Spider-Man(2002) [1]. We model the swing as an elastic pendulum. This means we can incorporate both the traditional swinging motion, but also the elastic nature of his webs. The aim of this paper is not necessarily to describe the scene with 100% accuracy, but rather to see if this model is a good way to describe that type of swing.

Theory

We can derive the equations of motion for an elastic pendulum by first constructing the Lagrangian for the system, to describe it in terms of its coordinates and their time derivatives. However, this paper is about forming a computer model, not deriving them, so we just state them, assuming the swing is only in 2 dimensions [2]:

$$\ddot{r} = (L_0 + r)\dot{\theta}^2 + g\cos\theta - \frac{k}{m}r \tag{1}$$

$$\ddot{\theta} = -\frac{g}{L_0 + r}\sin\theta - \frac{2\dot{r}\dot{\theta}}{L_0 + r} \tag{2}$$

Where g is acceleration due to gravity, the dots represent differentiation with respect to time, and the rest of the notation is described in Figure 1. We also note the equations do not take into account any drag or damping forces, and assume the spring has no mass. θ is defined so it is $0°$ when vertical, and increases anticlockwise.

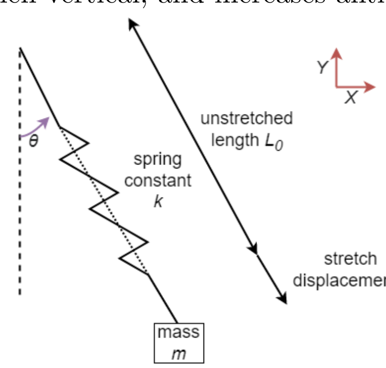

Figure 1: The nature of an elastic pendulum. It is essentially a traditional pendulum, but with the rope replaced with a spring (shown by the zigzag line), which we assume takes up the whole length. Inspired by [2].

Model

We create our model in Python using the equations of motion outlined in Eqs.(1) and (2), and integrate using a Python function. Before we apply it, we perform a couple of test runs to make sure it is behaving how we expect. For this test we choose natural units ($L_0, g, k, m = 1$) as these constants are just scaling factors for real life. One simple test is to set the initial angle to $\theta = 0°$, as this would remove all the pendulum motion so we would expect to see a vertical line, as θ can't change. Another test is to start it with some small angle (e.g. $\theta = 15°$), and we would expect to see it both bounce and oscillate, but never go above $15°$ as it shouldn't gain energy if we give it zero initial velocity.

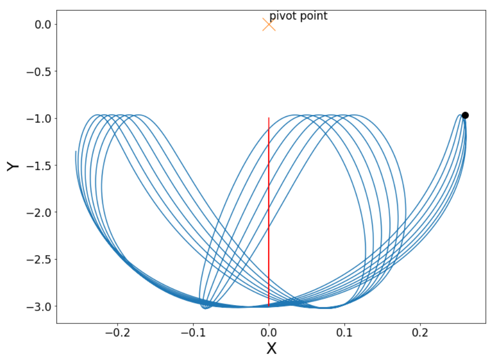

Figure 2: Test graphs for our model. Red: $\theta = 0°$, and it behaves like a normal mass-spring system. Blue: $\theta = 15°$ and it behaves like a spring-pendulum combination. The black dot shows the starting position ($15°$ although it is distorted by the scale). The line tracks the movement of the mass on the end.

Applying the model

In order to apply the model we need values for the constants. We chose a typical human mass $m = 70$ kg, and $g = 9.81$ ms^{-2} as usual. We assume this doesn't change significantly over the trajectory. The spring constant is given by:

$$k = \frac{F}{\Delta L} = \frac{mg}{r} \tag{3}$$

Where $\Delta L = r$ is the extension when he is at rest vertically. Comparing this to the height of buildings in the scene [1] we find $k \sim 30$ Nm^{-1}. Also in the scene, he appears to shoot the web when he is roughly level with the pivot point (the crane in this case), so we assume an initial $\theta = 90°$, and the distance away is approximately one building width which we assume is $L_0 \sim 50$ m.

Results & Discussion

Putting these numbers in the model and only running for the time before he lets go, we produce Figure 3. This trajectory seems to quite closely follow what is shown in the scene [1]. One of the key points is that he passes closely around the pivot, which we have achieved here, and that he has some velocity taking him up afterwards. Watching the scene [1], he approaches the swing with some initial velocity. In order to get this trajectory, we had to give him initial velocities of $\dot{r} = 5$ ms^{-1} and $\dot{\theta} = 1$ rads^{-1}. Whilst we cannot explicitly prove these are correct in this paper, they don't seem wildly inaccurate for how fast he appears to be moving in the scene.

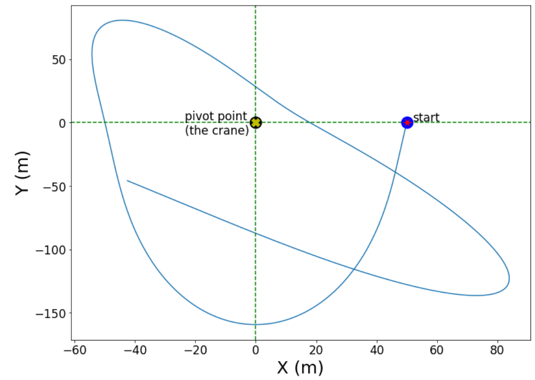

Figure 3: Trajectory of the Spider-Man swing

Conclusion

We have used the differential equations for an elastic pendulum to model the swing trajectory of a particular scene from Spider-Man(2002) [1]. We find that for sensible initial parameters this model holds quite well, hence it is a reasonable model to use to plot this type of swing. In the future, we may wish to compare this to another scene and see if we would get similar results also.

References

[1] Director: Sam Raimi. *Spider-Man (2002)*. Production companies: Columbia Pictures, Marvel Enterprises, Laura Ziskin Productions. Link to scene `https://youtu.be/5n8bMuZGipU`, 0:40. Accessed [04/11/22].

[2] David Morin. The Lagrangian Method. `https://scholar.harvard.edu/files/david-morin/files/cmchap6.pdf`, 2007. Accessed [04/11/22].