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A5_7 Laser Pool

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Abstract

In this paper we ask the question of how feasible the concept of a laser pool cue would be. The laser is initially considered to be of comparable diameter to that of the ball, and is compared to a more focused beam. The geometry of the system is considered in detail, calculating how much photon momentum is transferred to the ball. We find the required laser power is approximately 1×10^{11} W for the wide laser, and 2.5×10^{10} W for the focused laser.

Introduction

Futuristic media makes frequent use of light-based objects, such as the "lightsaber" in Star Wars, which have a moderate diameter. This prompted the thought of using the momentum of light to strike a pool ball, as if it was struck by a cue. The effect that parallel light rays have when bouncing off the ball from one direction is calculated, and the result is used to provide a necessary power value for the laser, using some assumptions. The surface of the ball is assumed to be perfectly smooth and reflective for simplicity, and friction is ignored.

Calculations

The first stage of this problem is to determine the component of the reflected photon momentum in the original axis of travel (defined as the $+x$ direction). We assume parallel photons striking the entirety of one face of the ball. Force can be given by $F = \mathrm{d}p/\mathrm{d}t$, or the rate of change of momentum. Considering an annulus of radius $R\sin\theta$ (where θ is the angle of incidence and R is the radius of the ball) and width $Rd\theta$, the incident perpendicular power within this area is

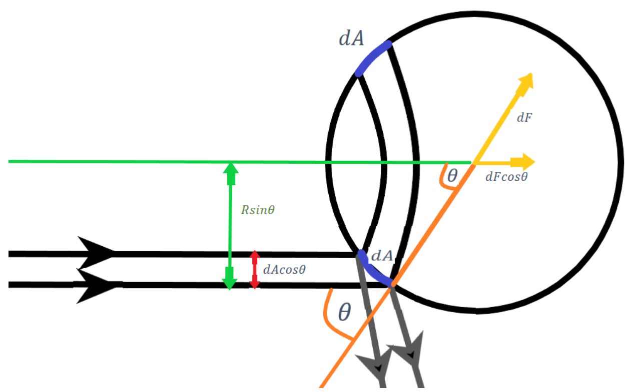

Figure 1: A diagram demonstrating the quantities that have been used to calculate the force on the ball. The two beams are intended to demonstrate the edge of a given small annulus which is being integrated over.

given by $dP = IdA\cos\theta$, where I is the intensity of the light per unit area in Wm^{-2}. Photon momentum $p = E/c$ (where E is the energy of a photon and c is the speed of light) and $I = E/(At)$ is the energy per unit area per second. By rearranging the latter equation for E and substituting into the momentum equation, the force can be found to be $F = 2IA/c$, where the 2 arises from the twice as high momentum change for a reflecting photon.

$$dF = \frac{2I}{c}(dA\cos\theta)\cos\theta. \quad (1)$$

The second factor of $\cos\theta$ arising from the light ray being deflected by 2θ, as opposed to being absorbed. The area of the annulus, dA, is given by $dA = 2\pi R\sin\theta \times Rd\theta$. The force exerted on the annulus is then

$$dF = \frac{4\pi IR^2}{c}\cos^2\theta\sin\theta d\theta. \quad (2)$$

θ can range between 0 and $\pi/2$, thus the total force on the ball in the x direction (as forces in the other axes are zero by symmetry) is given by

$$F = \int_0^{\pi/2} dF\cos\theta, \quad (3)$$

or substituting in dF,

$$F = \frac{4\pi IR^2}{c}\int_0^{\pi/2}\cos^3\theta\sin\theta d\theta. \quad (4)$$

Using the substitution $u = \cos\theta$, eq. 4 becomes

$$F = \frac{4\pi IR^2}{c}\int_1^0 -u^3 du, \quad (5)$$

resulting in a final answer of

$$F = \frac{\pi IR^2}{c}. \quad (6)$$

Application

Regulation pool balls have a diameter of $D = 5.715$ cm, and a mass of between 156 g and 170 g [1]. Here we take $m = 160$ g. Assuming the pool ball moves at 2 ms^{-1} after being struck [2], the ball would obtain a momentum of $0.160 \times 2 = 0.32$ kgms^{-1}. Again approximating the contact time of the cue/ball collision as 1 ms [3], the force on the ball is $0.32/0.001 = 320$ N. Rearranging eq. 6 for I yields

$$I = \frac{Fc}{\pi R^2}, \quad (7)$$

and substituting our values in produces a required intensity of approximately $4\times10^{13}\mathrm{Wm}^{-2}$. The required laser power for this application is therefore 1×10^{11} W.

A Simpler Calculation

By reducing the contact radius from that of the entire ball to a single point, a maximal proportion of the light's momentum will be transferred to the ball's horizontal momentum. The answer becomes significantly simpler, as the ball's geometry no longer needs to be considered. Here, the force on the ball is simply $F = 2IA/c$. Using this simplification, the required power of the laser becomes 2.5×10^{10} W, or a quarter of our original calculation.

Conclusion

As can be seen, the calculated values are absurd. They are far stronger than the strongest continuous wave lasers, such as the MIRACL laser developed by the US navy in the 1980s [4], which is ten thousand times weaker. As we have assumed perfect reflection, actual balls would not quite behave in the same way. A non-perfect reflecting surface and friction between the ball and table would cause the ball to absorb the even more extreme amounts of energy than calculated here, most likely vaporising the ball instantly.This method is evidently not a good idea for playing such games as pool or snooker, and we recommend using the usual cue.

References

[1] `https://wpapool.com/equipment-specifications/` [Accessed 28/10/22]

[2] `https://billiards.colostate.edu/faq/speed/typical/` [Accessed 28/10/22]

[3] `https://billiards.colostate.edu/faq/cue-tip/contact-time/` [Accessed 28/10/22]

[4] `https://www.globalsecurity.org/space/systems/miracl.html` [Accessed 31/10/22]