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A4_6 Longest Candle Alarm

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Abstract

In this paper, the longest amount of time a candle could take to burn is calculated using a previously determined burn rate for a taper candle and by finding the tallest a candle could possibly be. This amount of time was found to be approximately 5.92 years for a candle of height 1100 m, restricted by the availability of oxygen in the surrounding atmosphere.

Introduction

Prior to the development of spring-driven clocks in the 15th Century, time was kept through other means, including the burning of a candle with notches added at regular intervals to indicate how many hours have passed since its lighting. Another use of the form of timekeeping involved nails being added along the length of the candle, such that it would fall after a set amount of time and fall onto a metal plate to act as a rudimentary alarm clock. Naturally the primary limitation of this method is that the alarm duration can only last as long as the height of the candle used allows. As such it seems sensible to ask what is the longest duration an alarm of this nature could last on Earth?

Theory

The initial challenge we encounter when approaching this problem is that a candle's ability to burn is heavily dependent on the oxygen concentration of its environment. Obviously as an extension of this there is also then a maximum altitude for burning a candle, above which a candle could no longer be burnt. To find this height we can use the Barometric Formula [1] for air density, shown in Eq.(1),

$$\rho = \rho_0 \left[\frac{T_0}{T_0 + (h - h_0)L}\right]^{1 + \frac{gM}{RL}} \tag{1}$$

where ρ is air density, T is temperature, h is altitude, L is the temperature lapse rate, g is acceleration due to gravity, M is molar mass, R is the molar gas constant, and a subscript zero denotes a variable's value at sea level. This and the fact that Oxygen concentration changes proportionally with air density linearly as in Eq.(2),

$$\Omega = \Omega_0 \frac{\rho}{\rho_0} \tag{2}$$

where Ω is Oxygen concentration. Substituting Eq.(1) into Eq.(2) and rearranging for h gives Eq.(3) below.

$$h = h_0 + \frac{T_0}{L}\left(\left(\frac{\Omega}{\Omega_0}\right)^{-\frac{1}{1+\frac{gM}{RL}}} - 1\right) \tag{3}$$

A candle that burns at a constant rate loses height at a constant rate B ms^{-1}. One material that could have been used in these old fashioned alarm clocks is stearin, obtained from animal fats and vegetable oils, which for a standard taper candle diameter of 2.22 cm at sea level has a rate $B_0 = 2.31$ cm hour^{-1} [2]. While this burn rate is naturally affected by oxygen concentration, its effect on the burning of candle wax has been shown to have a linear relationship [3], e.g.

$$B = B_0 \frac{\Omega}{\Omega_0} = B_0 \frac{\rho}{\rho_0} \tag{4}$$

Since this burn rate can be defined, with t representing time, as

$$B = \frac{dh}{dt} \tag{5}$$

We can manipulate this relationship and substitute Eq.(4) to find that

$$t = \int \frac{1}{B} dh = \frac{1}{B_0} \int_{h_0}^{h_{max}} \frac{\rho_0}{\rho} dh \tag{6}$$

where ρ is the same expression given in Eq.(1) and h_{max} is the maximum altitude to be found in the analysis.

Results

The minimum oxygen concentration required for a candle wick to burn is 17.8% [4]. Using the set of values, $h_0 = 0$ m, $T_0 = 288$ K, $L = 0.0098$ Km^{-1} [5] , $\Omega = 17.8\%$, $\Omega_0 = 21\%$, $g = 9.81$ ms^{-2}, $M = 0.0290$ kgmol^{-1} and $R = 8.314$ Nmmol^{-1}K^{-1}, a value of 1100 m can be obtained from Eq.(3), as the highest altitude where a candle can therefore be burned.

The definite integral shown in Eq.(6), now using our new value for $h_{max} = 1100$ m, can be evaluated to 1200 m (the full integral is far too long to include here so will be excluded for convenience). Therefore the total time for the candle to burn is this value divided by B_0, as shown in Eq.(6), resulting $t = 51900$ hours $= 5.92$ years.

Discussion

Several things could be taken into consideration to further the accuracy of this result, such as the fact that a colder ambient temperature as you move up the atmosphere will unavoidably cause the candle wax to melt, and hence be burnt off, at a slower rate such that the total burn time of 5.92 years will increase. However, as the temperature lapse rate quoted, $L = 0.0098$ Km^{-1}, is expected to be mostly constant on the scale we have found to be working in, the temperature change from sea level to h_{max} is only approximately 11°C and so it was decided that it would not have a significant affect on the time taken to melt and burn the candle wax.

The wind would also of course affect the temperature of the wax even further, but for the sake of this paper we have assumed the candle experiences no wind or external force. This is simply because it would defeat the whole idea since it would cause the flame to be extinguished, as well as likely causing the candle to topple over due to its high center of mass relative to the diameter of its base.

The value we have obtained here is very likely to never have an impact on real life since people are unlikely to require a candle to burn for 5.92 years, let alone construct a taper candle of height 1100 m.

Conclusion

In conclusion, the longest physically possible candle alarm clock, though likely not the most physically viable, takes up 1100 m of height and lasts 5.92 years in total. So while modern technology can fulfil the need for an alarm clock of this duration much easier, if you are feeling old fashioned you can start a really tall alarm clock when this paper was written and be alerted just in time for Halloween 2028!

References

[1] `https://en.wikipedia.org/wiki/Barometric_formula` [Accessed 30/11/2022]

[2] Wayman, J. Burn, Baby, Burn!, p2, October 2022 [Accessed 30/11/2022]

[3] `https://bit.ly/3ESeZY3` [Accessed 30/11/2022]

[4] `https://bit.ly/3TcP5CF` [Accessed 30/11/2022]

[5] `https://en.wikipedia.org/wiki/Lapse_rate` [Accessed 30/11/2022]