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A2_1 Extreme Football Curves

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Abstract

In this paper, we investigate the conditions needed to kick and bend a ball, to form a complete loop. We compare our model to one of the most documented kicks in history, Roberto Carlos' famous free kick against France. We find that our model is in agreement with Carlos' kick, but from our analysis of the initial conditions, it is not possible to form a complete loop with our model.

Introduction

It is with tremendous practise and talent that footballers are able to curve a football with: top, back and side spin, to score goals that football fans would dream of scoring. Arguably the most famous kick in all of football is Roberto Carlos' free kick against France. The speed, curve and accuracy is legendary. We are investigating the universe where a divine power gave Carlos much more power and caused his free kick to miss, with the ball returning directly back to him.

Equations

To start, we will need to setup a model, where we can input a set of initial conditions and it will output the position of the ball. We will start by evaluating the forces that act on the ball:

$$\mathbf{F} = \mathbf{F}_g + \mathbf{F}_d + \mathbf{F}_L \tag{1}$$

Where $\mathbf{F}_g$, $\mathbf{F}_d$ and $\mathbf{F}_L$ are the forces due to gravity, drag and the Magnus force respectively. The Magnus force results from the pressure difference between the opposite ends of the ball, created by the spin of the ball. Evaluating each force, we get:

$$\mathbf{F}_g = m\mathbf{g} \tag{2}$$

$$\mathbf{F}_d = -\frac{1}{2}\rho A C_D |\mathbf{v}|\mathbf{v} \tag{3}$$

$$\mathbf{F}_L = \frac{1}{2}\rho A C_L |\mathbf{v}|\mathbf{v} \tag{4}$$

Where m is the mass of the ball, ρ is the density of air, A is the cross-sectional area of the football, C_D and C_L are the drag and lift coefficients and $\mathbf{v}$ is the velocity of the ball. We will be using the values of $m = 0.4$ kg, $r = 0.11$ m [1],(the radius of the ball), and $\mathbf{g}$ is acceleration due to gravity. We have simplified these equations, by assuming that the spin-axis of the ball is always orthogonal to the x-y plane.

The overall force is simply $\mathbf{F} = m\ddot{\mathbf{R}}$, so putting this, equations 2, 3 and 4 into 1, and splitting $\ddot{\mathbf{R}}$ into 3 differential equations, (one for each component of the acceleration, where $\ddot{\mathbf{R}} = (\ddot{x}, \ddot{y}, \ddot{z})$), will give us:

$$\ddot{x} = -|\mathbf{v}|\frac{\rho A}{2m}(C_D\dot{x} + C_L\dot{y}) \tag{5}$$

$$\ddot{y} = -|\mathbf{v}|\frac{\rho A}{2m}(C_D\dot{y} - C_L\dot{x}) \tag{6}$$

$$\ddot{z} = -|\mathbf{v}|\frac{\rho A}{2m}C_D\dot{z} - g \quad (7)$$

Now we need to figure out values for C_D and C_L. These are tough to determine but an experiment done yields C_D and C_L is roughly approximated with equation 9:

$$C_D = 0.413C_L^{0.306}[2,\ 3] \quad (8)$$

C_D is valid for velocities higher than $v_c = 12.2$ m s^{-1}[2] and $C_L > 0.05$, otherwise C_D would vary differently. However, almost all of the speeds in the model are greater than $|\mathbf{v}| > v_c$ and $C_L > 0.05$.

$$C_L = \frac{r\omega_0}{|\mathbf{v}|}[2,\ 3] \quad (9)$$

Results

Equations 5 to 7 have to be solved analytically, which was achieved by converting each equation into 2 first order differential equations using the substitutions: $\dot{R}_i = u_i$ and using the *odeint* package in python. The starting conditions used are based on R.Carlos' free kick, with the initial conditions of: $\mathbf{R}_0 = (0, 0, 0)$ m, $\dot{\mathbf{R}}_0 = (10.1, 35.1, 10.5)$ m s^{-1} and $\omega_0 = 88.0$ rad/s[4], with ω being the angular velocity.

Figure 1 shows some of the possible curves, predicted from our model. The values are chosen, for their intriguing results. The blue and red curve have, $\omega = 3\omega_0$ and $\omega = 7\omega_0$ respectively, while the yellow curve is an extreme example, with $\omega = 20\omega_0$ and $v_y = 20v_{y_0}$, (v_y being the R_y velocity). We can see that the yellow curve forms a spiral, resulting from the high angular velocity of the ball. The red curve almost creates a full loop, but just falls short from the origin, due to the drag of the ball decreasing the velocity.

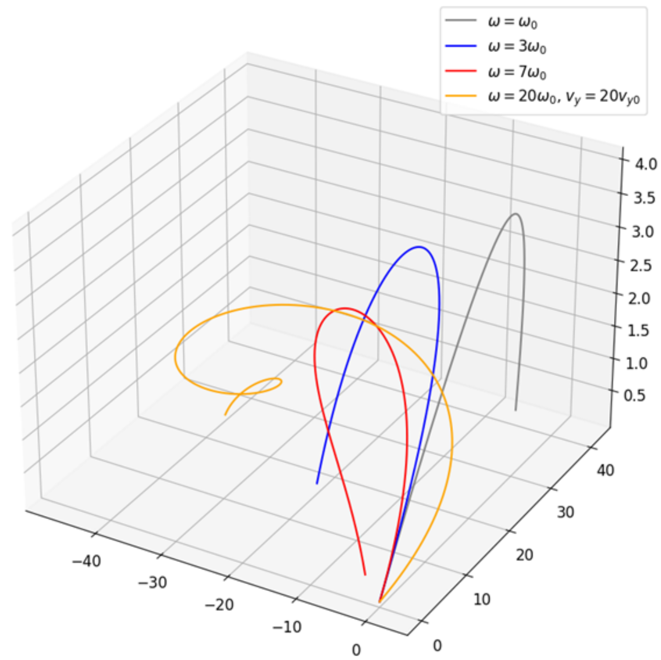

Figure 1: Plots of R.Carlos' free kicks, with differing initial conditions, highlighted by the legend. The Gray Curve represents the true R.Carlos' kick, with the goal being $y = 35$ m[4] away.

Conclusion

Given the conditions of the model, we were unable to find the set of initial conditions that would allow the ball to form a complete loop. Whilst a human may not be able to create curves as seen in Figure 1, it is possible that a machine shooting a ball could. A more thorough analysis of the initial conditions is needed, to see if it would be possible. The current state of the model suggests that it would not be possible. The simplified equations of 5 to 7 can be expanded upon, to account for a varying spin-axis of the ball. Also, one could use a better method to solve the end differential equations to make them more accurate.

References

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