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A5_4 Sending an Arrow to a Kilometer Away

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Abstract

Firing an arrow as far as possible has been a topic discussed for a long time, mainly during times when single piece wooden bows were the norm, and archery was mainly a form of combat. This paper answers what peak draw weight would be required, in order to launch an arrow to a kilometer away. We found that the peak draw weight required would be 578 N.

Assumptions

In order to carry out this calculation, we have made a few assumptions, since we are firing an arrow, which is a thin and narrow object we are assuming air resistance is negligible. We are also assuming that the draw force curve of the bow is linear. That from brace height (the distance of the string from the bow when not drawn) to full draw, the draw weight increases at a constant rate.

We are also assuming that this is an average sized archer with a draw length of 71.1 cm (converted from 28 inches) [1], and that the bow that we are using is approximately 80% efficient [2].

For the arrow mass calculations we are assuming that the mass of the fletching and nock is negligible.

Methodology

The first part of this calculation was to find out the velocity of the arrow required for it to travel a kilometer. We started by treating the arrow like a projectile in 2D motion, where the only force acting on the arrow once launched is its weight, vertically downwards. The two equations (1) and (2) below resolve the velocity into its vertical and horizontal motion, v_0 being the initial velocity, v_{0x} , v_{0y} the horizontal and vertical components of the velocity and θ the launch angle.

$$v_{0x} = v_0 \cos(\theta) \tag{1}$$

$$v_{0y} = v_0 \sin(\theta) \tag{2}$$

Since we said that the only force acting on the arrow after launch is the weight, there is no horizontal acceleration and a vertical acceleration of $-g$. Therefore the horizontal and vertical displacements can be written as (3) and (4), where s_x and s_y are the horizontal and vertical displacements.

$$s_x = v_0 t \cos(\theta) \tag{3}$$

$$s_y = v_{0y} t - \frac{gt^2}{2} \tag{4}$$

Since we want to use the maximum distance of the arrow, with (4) s_y can be set equal to 0. The non-zero result of t can be substituted into (1) giving us the following result.

$$R = \frac{v_0 \cos(\theta)(2v_0 \sin(\theta))}{g} \tag{5}$$

$$v_0 = \left(\frac{Rg}{\sin(2\theta)}\right)^{1/2} \tag{6}$$

In equation (5), R is defined as the range, which is the maximum horizontal distance of the arrow, using the trigonometric identity $2\sin(x)\cos(x) = \sin(2x)$, and rearranging this equation for initial velocity, it can be written in the form (6). From this we can see that the range will be at maximum when the angle is 45°, using the value of g as 9.81 ms^{-2} and the range as 1000 m the velocity required is 99.0 ms^{-1}.

The next stage of the calculation is the mass of the arrow, for this we will assume the arrow has a length of 71.1 cm (28" [1]). The mass of the arrow was calculated to be 23.2 grams by adding the mass of the shaft [3] and mass of the tips [4], (shaft mass per cm multiplied by draw length).

$$K = \frac{mv_0^2}{2} \tag{7}$$

Equation (7) is the equation for Kinetic energy K, we now know the mass of the arrow m and the launch velocity required, using that, the arrow would need 114 J in order to reach a kilometer, we also know from [2] that a modern recurve bow is approximately 80% efficient, which means that the bow would need to store 143 J at full draw.

$$W = Fd \tag{8}$$

The final stage of this calculation is to find the peak draw weight required, using equation (8), W is the work done which is equal to the energy stored by the bow, the maximum of force F is our peak draw weight and distance d equal to the draw length - the brace height (49.5 cm). The draw length is the full length from the bow to where the archer draws to, which we are using as 71.1 cm. The brace height is the distance from the string to the bow when not drawn, which for an average bow is 21.6 cm (converted from inches [5]). The reason we are using $d=$ draw length − brace height is because there is no horizontal force acting on the string of the bow at brace height, the archer is only doing work on the bow from the distance of the brace height to their draw length. Since the force exerted by the archer increases linearly with the distance, the equation for the peak draw weight D_{wmax} is given below (9). The reason for the factor of 2 being that the draw weight starts at 0 and increases linearly with d.

$$D_{wmax} = \frac{2W}{d} \tag{9}$$

The final answer for D_{wmax} is 578 N.

Conclusion

Using these calculations we have found a value for a peak draw weight of 578 N, given this information, it is highly unlikely that an arrow will ever be accidentally launched that far, it is not something required to consider when setting up an archery range. Given that a majority of top recurve archers draw bows of around 222 N [6], this value is more then double that, therefore we can also conclude that it is highly unlikely that an archer will be able to launch an arrow to a kilometer away.

References

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