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Abstract

We calculate the plausibility of a method of propulsion for space travel at relativistic velocities. This propulsion utilises the momentum from photons produced through particle-antiparticle annihilation. We outline the key physics involved in this method of propulsion and determine it is physically unfeasible. We calculate that a mass of 201,000 kg of ^{22}Mg is required to produce enough positrons to annihilate into photons to accelerate a 10 kg spacecraft up to 0.5c.

Introduction

It is common in science fiction that spacecraft travel at relativistic velocities at significant fractions of the speed of light. This would require a powerful source of propulsion. Antimatter is sometimes quoted as a fuel source for these fictional spacecraft (e.g. in Star Trek).

When particles interact with their antiparticle counterparts they annihilate and photons are produced. In order to conserve momentum two photons are produced travelling in opposite directions with equal kinetic energy. The combined kinetic energy of these photons is equal to the total energy of the original particle-antiparticle pair. It is theoretically possible that the momentum imparted to these photons can be used to accelerate a spacecraft from rest in space.

This could be done using a perfect mirror that reflects half of the photons out the back of the spacecraft. As momentum must be conserved these reflected photons would impart equal momentum on the craft.

Annihilation Energy

We must calculate the momentum of the photons by first calculating the annihilation energy. The most common particle-antiparticle interaction on Earth is the interaction between an electron and a positron [1]. For simplicity, we are assuming that these particles interact at rest with no kinetic energy. Therefore, only the rest energy of the particles need be considered. This negligence of the kinetic energy can also be used as a lower limit of the spacecraft's velocity as any kinetic collisions would give rise to more energetic photons. The rest mass of an electron and positron is 0.511 MeV each.

$$p_{(photon)} = \frac{E}{c} \tag{1}$$

Using the equation above, this means that the photons will have a momentum of 2.73×10^{-22} kgms^{-1}. One of these photons will be incident on the mirror and impart its momentum onto the spacecraft, increasing its velocity. We must first calculate the momentum required for a spacecraft to reach a given speed.

$$p_{(craft)} = m_{craft}v \tag{2}$$

It is assumed that the fuel-less mass of the spacecraft is 10 kg and is travelling at $0.5c$. This means that its momentum would be 1.50×10^{9} kgms^{-1}. Using simple division we calculate that 5.49×10^{30} photons must be incident on the mirror to cause this momentum change in the spacecraft. This means this number of positrons would also be needed for the interaction to occur.

Positron Production

A source of electrons is comparatively easy to obtain. However, a source of antiparticles like the positron is more difficult to obtain. The most common way of creating antiparticles is through radioactive decay. Magnesium-22 is a relatively low-mass, unstable isotope with a short half-life that decays creating a positron [2].

$$^{22}Mg \rightarrow ^{22} Na + \beta^{+} + \nu_e \tag{3}$$

This isotope has a half-life of 3.87 s [2]. The decay product ^{22}Na has a much longer half-life of 2.6 yrs [3], so the ^{22}Na decays will be ignored in this case. Therefore, we require 5.49×10^{30} ^{22}Mg atoms. This would correspond to a mass of 201,000 kg of magnesium. This was obtained by assuming 1 atom of ^{22}Mg would remain.

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{0.5}}} \tag{4}$$

By considering the half-life and the relationship between time and number of atoms (Eq. 4 above) we calculated how long it would take for the entire ^{22}Mg mass to decay:

$$t = t_{0.5} log_{0.5} \left(\frac{N}{N_0}\right) \tag{5}$$

$t =$ Time taken for decay
$t_{0.5} =$ Half-life of isotope
$N =$ Number of atoms remaining
$N_0 =$ Initial number of atoms

The time taken for all of these atoms to decay and each emit a positron is 395 s. This means the spacecraft would accelerate from rest to $0.5c$ in 395 s. This would result in an acceleration of 379,000 ms^{-2} or 38,700g ms^{-2}.

Discussion

From these calculations, we can conclude that this method of propulsion is unfeasible. The mass of 201,000 kg of ^{22}Mg for only a 10 kg spacecraft is paradoxically large. The acceleration is also too great and would destroy the spacecraft and any life onboard. This is because the method of positron production is too cumbersome and the rate of production is difficult to control. However, if there were a way of containing positrons in a strong magnetic field this would mean no isotope would need to be carried on the spacecraft, significantly reducing its mass. This method of positron containment would also mean the rate of annihilation could be controlled reducing the acceleration to a desired value. This could be done by releasing a stream of positrons and electrons into the 'combustion' chamber at a set rate. However, the energy required to generate the magnetic field required may be energetically unfeasible.

Conclusion

This method of propulsion is unfeasible as we have described it, due to the mass of the fuel source and the acceleration of the spacecraft. However, with modification to the method of positron production and containment, it is possible that this propulsion method could become feasible in the future. For now, it seems, this type of engine still remains only within the realms of science fiction!

References

[1] `https://www.britannica.com/science/annihilation` [Accessed 19 October 2022]

[2] `http://nucleardata.nuclear.lu.se/toi/nuclide.asp?iZA=120022` [Accessed 19 October 2022]

[3] `https://periodictable.com/Isotopes/011.22/index.full.html` [Accessed 19 October 2022]