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P3_6 Doctor Who - The Impossible Planet?

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Abstract

We investigate "The Impossible Planet" from Series 2, Episodes 8&9 of Doctor Who [1], a planet in "geostationary" orbit around a black hole. We find that it might not be impossible, by looking at the tidal forces on the planet and using simplified rotating black hole mechanics.

Introduction

In the eighth & ninth episodes of series two of Doctor Who [1], David Tennant's tenth doctor and companion Rose Tyler land on a sanctuary base on "the impossible planet" Krop Tor. They say this planet is in geostationary orbit around a black hole [1]. In this paper we will investigate just how impossible this planet is, by considering the orbital angular velocity of the planet in a region where it could exist, and comparing this to the limit of how fast a black hole can rotate.

Theory & Results

The main reason they give for why this planet is impossible is that it is near a black hole, but just being close to a black hole does not immediately mean destruction for the planet [2]. The planet would be destoyed when the tidal forces on the planet exceed the force of the planet's own gravity holding it together. We define the limit of how close the planet can get before being ripped apart as the Roche Limit r [3]:

$$r = \left(\frac{2m_1}{m_2}\right)^{\frac{1}{3}} R_2 \tag{1}$$

Where m is mass, 1 and 2 are the black hole and planet, and R_2 is the planet radius. We do not know values for the planet but the characters appear to experience Earth-like gravity so we will give the planet Earth-like characteristics. Also using a typical stellar-class black hole mass of $\sim 5M_\odot$ [4], we find the Roche limit is $r \sim 9.5 \times 10^8$ m. If we said that the planet was just stable, such that is it orbiting at this exact distance (and assume circular) we can find the orbital angular velocity:

$$\omega = \sqrt{\frac{Gm_1}{r^3}} \tag{2}$$

G is the gravitational constant, and we find $\omega \sim 8.8 \times 10^{-4}$ rads^{-1}.

We do not know any characteristics of the black hole, except that it must be spinning if the planet is in "geostationary" orbit - if that would even be the correct term for this. Here we just interpret it to mean that the black hole spin and planet orbit have the same angular velocity in their own frame. Given the black hole is spinning makes this situation very complicated. We will make a few simplifications. We essentially treat the outer event horizon as the "surface" of a normal sphere, and don't consider the rotation of spacetime caused by the black hole rotating. We'll say if this is a reasonable assumption later on. We also assume the black hole is not charged.

The angular velocity of a particle at the surface of the outer event horizon, Ω_H, (in natural units $c, G = 1$) can be said to be [5]:

$$\Omega_H = \frac{J}{2m_1(m_1^2 + \sqrt{m_1^4 - J^2})} \tag{3}$$

Where J is the angular momentum. We rearrange to make this the subject

$$J = \frac{4m_1^3\Omega_H}{4m_1^2\Omega_H^2 + 1} \tag{4}$$

If we convert Eq.(4) to SI units [6], use $m_1 \sim 5M_\odot$ like before, and use our interpretation of geostationary ($\Omega_H = \omega$) we find $J \sim 1.9 \times 10^{36}$ kgm^{2}s^{-1}. We also note that a black hole's angular momentum is limited in this way [5]:

$$J \leq \frac{Gm_1^2}{c} \tag{5}$$

This gives us a limit of 2.2×10^{43} kgm^{2}s^{-1}, and we can see this limit is larger than the actual J, hence the black hole could spin at this rate. We also know the planet would not be destroyed by tidal forces since the angular velocity we previously worked out was based on a region where the planet could survive.

We can also check if the assumption about treating it as a "normal sphere" was acceptable, by comparing the radius of the outer horizon to the Roche limit (how far our planet is). The outer horizon for an uncharged black hole is given by [5] (also converting to SI units [6]):

$$r_+ = \frac{Gm_1}{c^2} + \sqrt{\left(\frac{Gm_1}{c^2}\right)^2 - a^2} \tag{6}$$

Where $a = \frac{J}{m_1 c}$. We find $r_+ \sim 15000$ m. This is significantly less than the Roche limit (Eq.1) $r \sim 9.5 \times 10^8$ m where the planet is, so we think that this was a reasonable assumption, at least to first order.

Discussion & Conclusion

Using our simplifications of rotating black hole physics we have shown that the impossible planet might not be so impossible after all. We have shown that for a stellar-class black hole, a planet in a "geostationary" orbit, at a distance far enough away to not be destroyed by tidal forces, does not break the angular momentum limit (Eq.5). Whether or not this proves the impossible planet could exist in reality is another thing. We have ignored several other factors which would impact whether life would survive on the planet, such as radiation, temperature of the planet etc. Perhaps the sanctuary base located on this planet has some futuristic technology which is able account for all these problems. In future we may want to consider more advanced black hole physics, such as a better consideration of the black hole's angular velocity and inclusion of relativity, and see how this affects our conclusion.

References

[1] The Impossible Planet (S2E8) & The Satan Pit (S2E9). BBC, 2006. Director: James Strong, Writer: Matt Jones, Executive Producers: Russell T. Davies & Julie Gardner.

[2] Space.com - Rafi Letzter. Thousands of Earthlike 'blanets' might circle the Milky Way's central black hole. `https://www.space.com/black-hole-planets-blanets.html`, 2020. Accessed [12/10/22].

[3] Imke De Pater and Jack J. Lissauer. *Planetary sciences*. Cambridge University Press, Cambridge, 2nd ed.. edition, 2010.

[4] Space.com. Black holes: Everything you need to know. `https://www.space.com/15421-black-holes-facts-formation-discovery-sdcmp.html`, 2022. Accessed [12/10/22].

[5] P. K. Townsend. Black holes. `https://arxiv.org/abs/gr-qc/9707012`, 1997. Accessed [12/10/22].

[6] `https://en.wikipedia.org/wiki/Geometrized_unit_system`. Accessed [01/11/22].