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P6_2 Mike Drop: Calculating the stretch of rope

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Abstract

Young's Modulus is used to calculate the stretch of a material based on the force exerted and the initial length. In this paper, this principal will be used to calculate the predicted stretch of a sampled dynamic climbing rope due to falls of sequential heights and compare them to the advertised stretch specification. The model was found to provide an underestimate at the limit of the model, returning a stretch of 21%.

Introduction

The stretch of climbing ropes is often an important feature of the equipment, especially when considering the 'dynamic' ropes used for lead/traditional climbing. Falls during these disciplines tend to be considerably greater than those during indoor top-roping due to the position of the top anchor relative to the falling party. Lead/trad climbs move to consecutive top anchors as the climber progresses resulting in the top anchor being below the climber for much of the climb, opposed to top-roping where the anchor is always above the climber. These large falls can result in hitting the ground.

Method

The basis of the model used in this paper to calculate the stretch of rope is the equation for Young's Modulus.

$$Y = \frac{FL_0}{A\Delta L} \tag{1}$$

Where Y is the Young's Modulus, F is the force applied, L_0 is the initial length of the cord, A is the cross-sectional area of the cord, and ΔL is the change in length due to applied force or length stretched. Investigation of the stretch requires a rearrangement.

$$\Delta L = \frac{FL_0}{AY} \tag{2}$$

Where AY can be substituted for the bulk property k, the spring constant of the rope[1]. This value is quoted in the technical specifications of the ropes found on the product pages [2]. We chose to sample the KARMA 9.8mm rope manufactured by Beal due to its fairly standard diameter and construction technique as well as the manufacturer being reputable [3].

To calculate the force applied multiple equations were used. The first to calculate the velocity at the anchor point, where S_0 is the distance above the anchor, a is acceleration due to gravity, and v_n is the velocity at point n ($n = 0$ at start, then $n = 1$ etc.).

$$v_1^2 = v_0^2 + 2aS_0 \tag{3}$$

This results in $v_1 = \sqrt{2aS_0}$ as $v_0 = 0$. A second value t is required to convert this into a force. To aquire t we use $Eq.(4)$ derived from $SUVAT$.

$$t = \frac{2y}{(v_1 + v_2)} \tag{4}$$

Where y is a set distance bellow the anchor point over which deceleration occurs. As $v_2 = 0$ these values can be used to calculate the force of the fall, where m is the mass of the climber.

$$F = m\frac{dv}{dt} = m\frac{v_1}{\frac{2y}{v_1}} \tag{5}$$

Substituting $v_1 = \sqrt{2aS_0}$ from $Eq.(3)$ and F from $Eq.(5)$ into $Eq.(2)$ gives.

$$\Delta L = m\frac{v_1^2 L_0}{2yk} = m\frac{aS_0L_0}{yk} \tag{6}$$

This equation provides the basis for the model used. Substituting values in from manufacture data, setting values for L_0 and y and setting a to the gravitational constant, the model was then run over a range of S_0 values between 0 and 2.

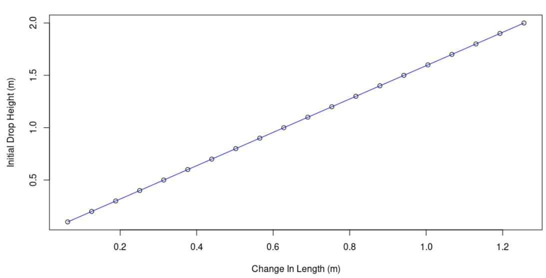

Figure 1: On the Y-axis S_0 is plotted against ΔL on the X-axis. The gradient can be expressed as $\frac{maL_0}{yk}$ this can be predicted from $Eq.(6)$

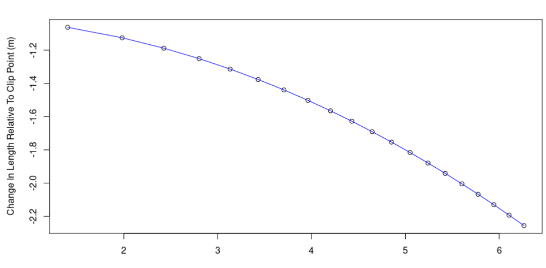

Figure 2: On the Y-axis $-(\Delta L + y)$ is plotted which is a value for displacement below the anchor point against v_1.

Figure 1 shows that for a drop of 2m a 6m section of rope stretches to a length of 7.255m which represents a ΔL of 1.255m. This is an elongation of slightly under 21% which is below the dynamic extension quoted in the KARMA ropes technical specification of 36%. this shows our model as stands provides an underestimate, however it does provide a result in a reasonable range.

Conclusion

The difference between our model and the extension stated by Beal could be due to a number of factors. For instance, the model has not taken into account the 'static stretch'(the stretch due to hanging a weight) this additional source of stretch would increase the overall force. This could be taken into account if the model were to be developed further.

Overall the model performed adequately to provide estimates of the stretch of a portion of rope when dynamically weighted, calculating a maximum stretch of 21% at the limit of the models test range.

References

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