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P3_1 A Water Powered Funicular Railway

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Abstract

We are investigating the weight of water needed to overcome frictional forces to power a funicular railway. We find that the amount of water required depends on the angle of the slope, and that angles below $\sim 28°$ are forbidden, and only above $\sim 35°$ do they become physically viable.

Introduction

A funicular railway is an efficient way to transport people up and down steep slopes. There are two trains running simultaneously, one going up and one going down, connected by cables, which run through pulleys at each end. In this way, they utilise the weight of the train moving down, to pull the other one up. You just need to overcome the friction in the system. This is usually provided by a motor but could alternatively come from adding water to the upper train, so it feels a greater force pulling it down than the lower train [1]. In this paper we will investigate the amount of water you would need to add to the upper train so that the funicular will work.

Theory

Figure:1 shows the force diagrams for the system. We define the mass as m and label each train 1 and 2 as shown. g is acceleration due to gravity and θ is the angle of the slope with respect to the horizontal ($0 \leq \theta \leq 90$). We also use the fact that static friction is the normal force multiplied by the coefficient of static friction (μ_s) [2]. We use static friction here as the system is not yet moving. Rolling friction is always less so once it's moving, the friction it experiences will always be less than what it has already overcome. We also neglect friction in the pulleys, air resistance and assume the cable mass is negligible.

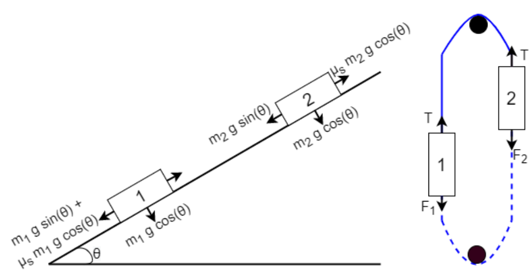

Figure 1: Force diagrams for our system. Left: side view with forces on each train due to itself. Right: top view with cable (blue) - the bottom cable (dotted) is just for support in the system. The tension in the cable & forces Eq.(1) and (2) are shown.

$$F_2 = m_2 g \sin\theta - \mu_s m_2 g \cos\theta \quad (1)$$

$$F_1 = m_1 g \sin\theta + \mu_s m_1 g \cos\theta \quad (2)$$

In order to move, the tension T pulling 1 up must be greater than the force pulling it down F_1. We construct Eq.(3) describing the forces on each train, and add them. This is fine, as if you straightened the cable (so a is in the same direction), that force diagram would be equivalent.

$$m_1 a = T - F_1 \ , \quad m_2 a = F_2 - T \tag{3}$$

$$(m_1 + m_2)a = F_2 - F_1 \tag{4}$$

Since the train is just about to move, $a = 0$. We move F_1 to the other side and write it all out:

$$m_2 g \sin\theta - \mu_s m_2 g \cos\theta = m_1 g \sin\theta + \mu_s m_1 g \cos\theta \tag{5}$$

We can take a factor of the mass out on each side, see that g cancels, and rearrange to:

$$\frac{m_2}{m_1} = \frac{\sin\theta + \mu_s \cos\theta}{\sin\theta - \mu_s \cos\theta} \tag{6}$$

If we assume each train has the same mass and passenger count (combined is "carriage mass"), we can find the mass of water required (m_w) on the second train, in units of carriage mass.

$$\frac{m_2}{m_1} = \frac{m_1 + m_w}{m_1} \implies \frac{m_w}{m_1} = \frac{\sin\theta + \mu_s \cos\theta}{\sin\theta - \mu_s \cos\theta} - 1 \tag{7}$$

Results & Discussion

Now we use this expression to calculate the mass of water required for different angles. A typical coefficient of static friction for train wheels on dry train tracks is $\mu_s = 0.5$ [3]. We will explore for all angles. We expect that smaller angles will require more water than larger ones.

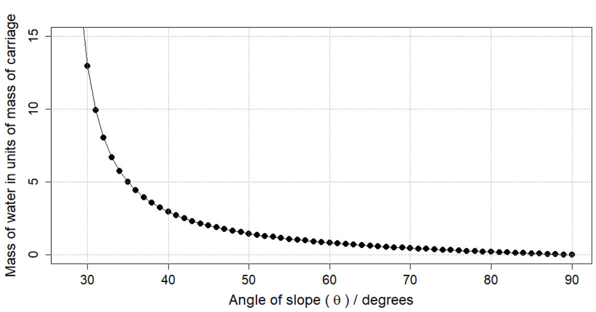

Figure 2: Graph showing how the mass of water required for movement varies with the slope angle.

One important thing to mention is that below $\sim 28°$ (the "absolute critical angle"), the mass of water comes out as negative. This is not physically possible - so we don't include it on Figure:2. It tells us that a system below that angle is also not physically possible. More of the weight of the water is pushing it into the track to cause friction than pulling it down the slope, so the train will never move. Although, just above this angle the mass of water which is required is much more than could realistically be carried on the train. If we said that a realistic amount that could be carried is 5 times the carriage mass, then our "realistic critical angle" is $\sim 35°$.

We find that (above the absolute critical angle), the mass of water required drops off as the angle increases. This is what we would have expected thinking about the problem intuitively. For a larger angle, more weight will be in the direction taking it down the track so there is less friction and less water is needed to overcome it.

Conclusion

We have found using water to power a funicular railway is only really viable for slope angles above $\sim 35°$. Below this it is either not physically possible, or would require more water than the train could carry. In the future we may also wish to consider the power that this water provides and see how this compares to the power of motors typically used to see which would be a lower power system for a specified angle. We may also wish to add back some aspects, e.g. pulley friction, and perform a more detailed analysis.

References

[1] `https://www.amusingplanet.com/2019/02/water-powered-funiculars.html` *Water Powered Funiculars*, Kaushik Patowary(2019) [Accessed 1 October 2022]

[2] P. A. Tipler & G. Mosca, *Physics For Scientists and Engineers with Modern Physics Sixth Edition* (W. H. Freeman and Company, New York, 2008).

[3] `https://the-contact-patch.com/book/rail/r1717-friction-between-wheel-and-rail` *R.1717 Friction between wheel and rail*, The Contact Patch(2014) [Accessed 2 October 2022]