Journal of Physics Special Topics
An undergraduate physics journal

P2_14 Gasolina

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December 12, 2021

Abstract

This paper determines the effects of gasoline and water compression on the travel time of a Bathyscaphe vessel as it descends to the bottom of the Mariana's Trench and rises to the surface again. The mean velocities of the vessel as it sinks and rises were calculated to be $v_{\text{avg sink}} = 0.65$ m s^{-1} and $v_{\text{avg rise}} = 1.44$ m s^{-1} respectively. The time taken to reach the bottom is 4 hours and 41 minutes; the time to rise to the surface is 2 hours and 7 minutes.

Introduction

The bathyscaphe vessel is analogous to an "underwater hot air balloon" where gasoline tanks provide positive buoyancy and air tanks and weights provide negative buoyancy. We aim to determine the effects of water and gasoline compression on the descent and rise times of the vessel described in our previous paper [1] on its round trip to the bottom of the ocean.

Theory

The bulk modulus equation for fluid compressibility can be rearranged into:

$$\Delta V = -\frac{\Delta P}{k} V_0 \tag{1}$$

where k is the bulk modulus, V_0 is the uncompressed volume of the fluid, and ΔV is the change in volume of the fluid in response to changing pressure, which is given by:

$$\Delta P = \rho_{0,w} g d \tag{2}$$

where $\rho_{0,w}$ is the density of uncompressed water, g is the acceleration due to gravity, and d is the depth from the surface. The change in volume for water, ΔV_w, and gasoline, ΔV_{g} can be found by substituting Eq. (2) into Eq. (1) and using the bulk modulus values of water and gasoline respectively. The new volumes of water and gasoline are given by:

$$V_w = V_{0,w} + \Delta V_w \tag{3}$$

$$V_g = V_{0,g} + \Delta V_g \tag{4}$$

The density values of compressed water and gasoline are given by:

$$\rho_w = \frac{V_{0,w}}{V_w} \rho_{0,w} \tag{5}$$

$$\rho_g = \frac{V_{0,g}}{V_g} \rho_{0,g} \tag{6}$$

As the water begins to compress, surrounding water will fill the empty spaces caused by the compression and so its volume will remain unchanged, $V_w = V_{0,w}$. Similarly, water will fill the empty spaces as the gasoline compresses which will leave the gasoline volume reduced and the water volume increased. Therefore, additional mass in the vessel needs to be accounted for which is given by the water density multiplied by the magnitude of the change in volume of the gasoline, $\rho_w|\Delta V_g|$, as this value is negative. The equation describing the density of the vessel as it sinks, Eq. (7), is adapted from our previous paper [1], with the values of constants from the same paper substituted in and simplified.

$$\rho_{\text{sink}} = \frac{16394 + 2\rho_{\text{Fe}} + V_{0,w}\rho_w + V_g\rho_g + \rho_w|\Delta V_g|}{107.2} \tag{7}$$

where ρ_{Fe} is the density of the iron weights on the vessel, ρ_w the density of the compressed water, and ρ_g the density of gasoline. The weights are released to begin the vessel's rise to the surface, which causes water to fill the empty spaces in the container previously occupied by the weights. The density as the vessel rises can therefore be found by replacing ρ_{Fe} with ρ_w in Eq. (7)

$$\rho_{\text{rise}} = \frac{16394 + (2 + V_{0,w})\rho_w + V_g\rho_g + \rho_w|\Delta V_g|}{107.2} \tag{8}$$

The equations describing the velocity of the vessel are adapted from our previous paper [2], with the values of constants from the same paper substituted in and simplified to be ≈ 30:

$$v_{\text{sink}} = \sqrt{\frac{30(\rho_{\text{sink}} - \rho_w)}{\rho_w}} \tag{9}$$

$$v_{\text{rise}} = \sqrt{\frac{30(\rho_w - \rho_{\text{rise}})}{\rho_w}} \tag{10}$$

where v_{sink} and v_{rise} are the velocities of the vessel whilst it is sinking and rising respectively. The duration of travel, t, is then given by d/v.

Results

The change in volume of gasoline and water as a function of depth is calculated by substituting Eq. (2) into Eq. (1). This is then substituted into Eq (3) and (4) to find the new volumes of water and gasoline against depth. The results are used to calculate the density of the compressed water and gasoline against depth using Eq. (5) and (6), which can be used to calculate the density of the vessel whilst sinking using Eq. (7) and whilst rising using Eq. (8). Using these results, the sinking and rising velocities can be calculated as a function of depth by substitution into Eq. (9) and (10).

The following values were used in our calculations: $d = 10984$ m, uncompressed densities of $\rho_{0,w} \approx 1022$ kg m^{-3} and $\rho_{0,g} \approx 800$ kg m^{-3}, initial volumes of $V_{0,w} = 9.93$ m^3 and $V_{0,g} = 89.4$ m^3 [3], and bulk modulus of water and gasoline as $k_w = 2.34 \times 10^9$ N m^{-2} and $k_g = 1.3 \times 10^9$ N m^{-2} respectively [4]. The resulting velocities as the vessel sinks and rises are plotted in Fig. 1.

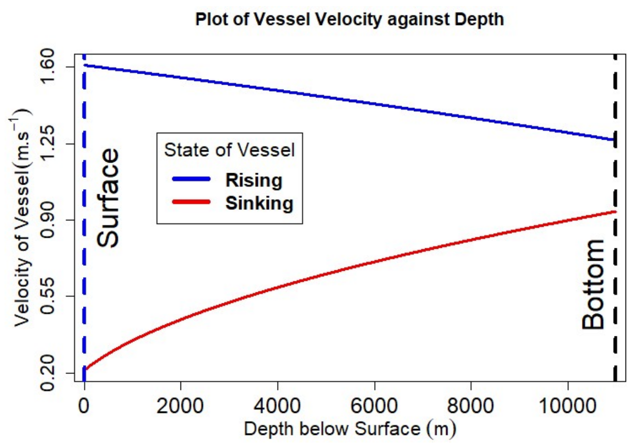

Figure 1: Graph of vessel's velocity against depth, with red denoting during descent and blue during ascent.

Conclusion

The mean values of the sinking and rising velocities were calculated to be $v_{\text{avg sink}} = 0.65$ m s^{-1} and $v_{\text{avg rise}} = 1.44$ m s^{-1}. The time taken to reach the bottom is then 4 hours and 41 minutes, whereas the time taken to rise to the surface again is 2 hours and 7 minutes.

References

[1] Murgatroyd et al, *P2_7 Underwater Hot Air Balloon*, PST 20, (2021)

[2] Murgatroyd et al, *P2_8 Journey to the Depths of the Earth*, PST 20, (2021)

[3] Murgatroyd et al, *P2_6 SubMariana*, PST 20, (2021)

[4] https://bit.ly/3G2FerZ [Accessed 1 December 2021]