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P2_11 Predictive Model of Baking Cakes

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Abstract

This paper outlines the creation of a computational predictive model designed to estimate the time to bake a cake based on the total summed mass of its ingredients. The model constructed showed expected behaviours, with the centre of the cake increasing in temperature asymptotically to the oven temperature and gave reasonable agreement with baking times when compared to various online recipes.

Introduction

Baking is a very common hobby and chocolate cakes are one of the most popular bakes in Britain [1]. The time a cake spends in the oven is very important – leaving it too long creates a dry, over-baked, and unenjoyable cake, while not leaving it long enough leads to an under-baked cake which may still contain raw batter in its centre. Despite this, recipes frequently give a number rounded to the nearest 5 minutes, or a large range of times, making it very difficult for a new baker to easily ensure that their cake is baked optimally. It is the aim of this paper to create a predictive model that allows new bakers to find a precise value of time for which to bake their cakes for optimum results.

Method

The equation for thermal current will serve as the basis of our predictive model for baking cakes:

$$I = kA\frac{\Delta T}{\Delta x} \tag{1}$$

where I is the thermal current due to conduction, k is the thermal conductivity, A is the cross-sectional area of the conductor, ΔT is the difference in temperature between the oven and the centre of the cake, and Δx is the distance over which the thermal current travels. If the cake batter is assumed to be spherical, the changing cross-sectional area can be considered by taking area elements, then integrating over all radii:

$$dI = k\frac{\Delta T}{\Delta x}dA \tag{2}$$

where $A = 4\pi r^2$, $dA = 8\pi r dr$ and Δx=R, which when substituted into Eq.(1) and integrated gives:

$$I = 4\pi kR(T_{oven} - T_{centre}) \tag{3}$$

with R being the total radius of the sphere. This can be modified to be in terms of mass and density, by using the volume equation of a sphere $V = \frac{4}{3}\pi R^3$ and the relationship between density and volume $\rho = m/V$ to give:

$$I = 4\pi k\left(\frac{3m}{4\pi\rho}\right)^{\frac{1}{3}}(T_{oven} - T_{centre}) \tag{4}$$

T_{centre} increases over time as the thermal current carries heat energy to it, causing the thermal current to decrease as a result. The equation $Q = mc\Delta T$ allows the temperature of the centre to be expressed as:

$$T_{centre}(t) = T_{centre}(t - \Delta t) + \frac{I\Delta t}{mc} \qquad (5)$$

Starting with T_{centre} equal to room temperature, the initial value of I is calculated using Eq.(4). These values, along with the temperature difference ΔT, can be substituted into Eq.(5) to find the increase in temperature of the centre, T_{centre} over a time period Δt. These new values are then substituted into Eq.(4) to update I before substituting back into Eq.(5). This process can be iterated over a desired time-frame to find the time it takes for the cake to finish cooking, when $T_{centre} = 99$ C [2].

Results

The value of k for cake batter can be assumed to be ≈ 1.532 W m^{-1} K^{-1} [3], the specific heat capacity can estimated to be 2587 J kg^{-1} K^{-1} [3], with a density of 0.95 kg m^{-3} [4]. The simulation was ran for various masses and compared to baking times given in recipes of equal mass.

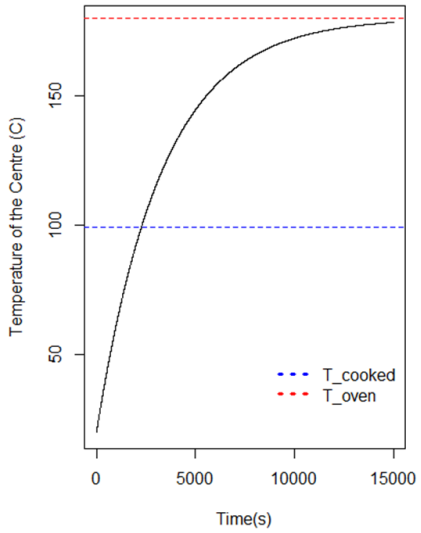

Figure 1: Simulated baking of a 1.483 kg cake in a 180 C oven

A 1.483 kg batter takes 34.6 minutes to bake according to the simulation, compared to 30-35 minutes given in a recipe [5], and takes 25.57 minutes for a 0.942 kg recipe, compared to the recommended 25 minutes [6].

Discussion

While not accounting for the expansion of the cake during the bake and assuming a spherical shape, the simulated model appears to make reasonable predictions for the time required to bake a cake based solely on its mass. Future investigation could include an attempt to incorporate the cake's expansion, or to modify the equation so that a cylindrical shape is used.

References

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