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P1_10 Solar propulsion

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Abstract

Solar sails and electric sails are proposed methods of spacecraft propulsion, they both use particles from the Sun to produce this thrust. But, to produce thrusts equivalent to modern ion engines (0.5 N)[4], at 1au a solar sail would need to be $54,900m^2$ and an electric sail would need to have $8.33 \times 10^6 m$ of wire, this makes both methods only practical for light spacecraft.

Introduction

As our experience exploring space improves, we have increased demands for our missions, but often the limiting factor to these missions is propulsion, such as NASA's Dawn mission taking 4 years to reach Vesta [3]. To solve this new propulsion techniques such as solar sails and electric sails have been proposed, both taking energy from the Sun and converting it into thrust for the spacecraft. This should allow new missions to shorten their timescales between launch and arrival or enable more ambitious missions. The main competitor to both solar sails and electric sails are ion thrusters as they have been used on long duration missions such as NASA's Dawn mission [3]. Current ion thrusters can achieve thrusts of 0.5N [4] so how large would a solar sail need to be and how much wire would an electric sail need to achieve the same thrust at 1 au?

Theory

Solar sails extract energy by reflecting photons from the Sun. Photons despite being mass less particles carry momentum, the magnitude of this momentum can be expressed as Eq. 1 [1]. Where p is the momentum, h is planks constant and λ is the wavelength of the light.

$$p = \frac{h}{\lambda} \tag{1}$$

As momentum must be conserved if a photon is absorbed by an object, it's momentum must be transferred to the object that absorbs the photon. Additionally, if a photon is reflected the interaction will also conserve momentum, but if we assume the photon is reflected along its original path, it will result in twice the momentum transferred compared to an absorbed photon.

Due to this, we can assume any surface that light from the Sun hits will experience a force acting on it, this can be known as Radiation pressure. To calculate the radiation pressure, we can use Eq. 2 [1] and Eq. 3 [1], where p_s is the pressure produced by the Sun I_x is the Sun's intensity and c is the speed of light.

$$p_s = \frac{I_x}{c} \tag{2}$$

$$I_x = \frac{L_{\odot}}{4\pi r^2} \tag{3}$$

Where $L_{\odot}$ is the luminosity of the Sun and r is the distance of the sail from the Sun. However, this assumes all the light is absorbed by the sail, if we assume some light is reflected the equation becomes Eq. 4 [1], as any reflected photons will contribute double the momentum of absorbed photons.

$$p_r = (1+R)\frac{I_x}{c} \tag{4}$$

Where R is the reflectance of the material used for the sail (the proportion of photons being reflected vs absorbed).

As this gives the pressure the sail will experience, we can calculate the total force on the spacecraft in Eq. 5 where A is the area of the sail.

$$F = p_r A \tag{5}$$

These calculations are covered in more detail in THE PHYSICS OF SOLAR SAILS [1]

Electric sails operate on a similar principle however rather than reflecting or absorbing photons, electric sails deflect plasma from the solar wind through large electric potentials.

The solar plasma is a neutral plasma consisting of both ions and electrons, moving at a roughly constant 400 km/s but with decreasing density as you move out through the solar system. If the plasma encounters a positively charged wire, positive ions in the plasma will be repelled, additionally electrons in the plasma will be attracted to the wires. This means if the wire is charged to a large enough potential it can reflect some positive ions back towards the Sun and "absorb" any momentum from the electrons. As shown in Janhunen et al (2007) [2] this can be used to derive Eq. 6 [2].

$$\frac{dF}{dz} = \frac{K m_p n_0 v^2 r_0}{\sqrt{exp[\frac{m_p v^2}{eV_0}\ ln(\frac{r_0}{r_w})] - 1}} \tag{6}$$

Where $\frac{dF}{dz}$ is the force per unit length of wire, n_0 is the undisturbed solar electron density, V_0 is the wires potential, r_w is the radius of the wire, e is the charge on an electron, m_p is the mass of a proton, r_0 is the radius from the wire where the potential has no influence, v is the speed of the solar wind and K is a constant that must be calculated.

Using Eq. 6, Janhunen et al (2007) [2] show the value of force per unit length is $\frac{dF}{dz} = (5 \pm 1) \times 10^{-8} N/m$ with a potential of 15 kV.

Results

For a solar sail at 1 au $I_x = 1,368 w/m^2$ [1] using $L_{\odot} = 3.83 \times 10^{26} w$ [1] and $r = 1.49 \times 10^{11} m$ [1] this gives $p_r = 9.1 \times 10^{-6} N/m^2$ [1] assuming maximum reflectance (R=1). Therefore, to give a force of 0.5 N the sail would need to be $54,900 m^2$ or if it was a square sail have sides 234m wide.

For an electric sail using the most generous estimate of $6 \times 10^{-8} N/m$ [2] for the force per unit length, the spacecraft would need $8.33 \times 10^6 m$ of wire in total, if the wire was separated out into 100 evenly spaced sections around the spacecraft each length would need to be 83 km long.

Conclusion

In conclusion, although harnessing energy from the Sun would allow for more ambitious missions any spacecraft using these technologies will likely need to be very small and light to allow solar sails and electric sails to propel them due to the very low thrust. Additionally, ion engines will likely still be preferred for larger slow acceleration spacecrafts.

References

[1] THE PHYSICS OF SOLAR SAILS https://ntrs.nasa.gov/api/citations/20030093608/downloads/20030093608.pdf [Accessed 01/12/21]

[2] P. Janhunen and A. Sandroos (2007) Solar wind electric sail propulsion, Bibcode: 2007AnGeo..25..755J

[3] https://solarsystem.nasa.gov/missions/dawn/technology/ion-propulsion/[Accessed 01/12/21]

[4] https://www.nasa.gov/centers/glenn/technology/Ion$_{P}ropulsion1.html[Accessed$01/12/21]