Journal of Physics Special Topics

An undergraduate physics journal

P2_8 Journey to the Depths of the Earth

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December 6, 2021

Abstract

This paper aims to determine velocity and travel time of a Bathyscaphe vessel as it descends to the bottom of the Mariana's Trench and rises to the surface again. Using the density of the vessel while submerging as $\rho_{\text{sink}} = 1100$ kg m^{-3}, the terminal velocity of the vessel is $v_{\text{sink}} = 1.47$ m s^{-1} and it will take 2 hours and 5 minutes to reach the bottom. Similarly, using the density of the vessel as it rises as $\rho_{\text{rise}} = 1020$ kg m^{-3} gives a terminal rise velocity of $v_{\text{rise}} = 0.235$ m s^{-1} and takes 12 hours and 59 minutes to reach the surface. The total travel time is 15 hours and 5 minutes.

Introduction

As previously found, the bathyscaphe vessel effectively acts as an "underwater hot air balloon" where the gasoline tanks are used to provide positive buoyancy, and air tanks and ballast weights are used to provide trim and negative buoyancy [1]. The aim of this paper is to determine the velocity and time taken for the vessel described in paper [2] to descend and ascend on its round trip to the bottom of the ocean.

Theory

For an object underwater, the net force experienced is given by:

$$F_{\text{net}} = W - F_B - F_D \tag{1}$$

where W is the weight of the vessel, F_B is the buoyant force, and F_D is the drag force. The weight of the vessel is given by:

$$W = V\rho g \tag{2}$$

where V is the volume of the vessel and ρ is the density of the vessel. The density whilst the vessel is submerging will be different than whilst it is rising and so the calculation for time and velocity will be done in two parts: during descent and ascent. The buoyancy force is given by the amount of water displaced by the object:

$$F_B = V\rho_w g \tag{3}$$

where ρ_w is the density of water, V is the volume of the vessel, and g is the acceleration due to gravity. Finally, the drag force is given by:

$$F_d = \frac{1}{2}C_D\rho_w A v^2 \tag{4}$$

where C_D is the drag coefficient, A is the cross-sectional area of the object in the direction of motion, and v is the velocity of the vessel. Importantly, during descent this will act in an upwards direction to oppose the motion of the vessel, however, whilst the vessel begins rising to the surface this will act in the opposite direction and become negative.

Assuming the object travels at its terminal velocity, the forces must balance and therefore:

$$W_{\text{sink}} = F_B + F_D \tag{5}$$

$$W_{\text{rise}} = F_B - F_D \tag{6}$$

The equations for velocity can be found by substituting Eq. 2, 3, and 4 into Eq. 5 and 6 and rearranging:

$$v_{\text{sink}} = \sqrt{\frac{2g(\rho_{\text{sink}} - \rho_w)}{C_D \rho_w} \left(\frac{V}{A}\right)} \tag{7}$$

$$v_{\text{rise}} = \sqrt{\frac{2g(\rho_w - \rho_{\text{rise}})}{C_D \rho_w} \left(\frac{V}{A}\right)} \tag{8}$$

The time taken to reach depth d can then be described by:

$$t = d/v \tag{9}$$

The volume is $V = 107$ m^3 and the density whilst submerging is $\rho_{\text{sink}} = 1100$ kg m^{-3}, and $\rho_{\text{rise}} = 1020$ kg m^{-3} whilst ascending. The water density in the trench is $\rho_w = 1022$ kg m^{-3} [2]. The cross-sectional area of the bathyscaphe can be found by approximating the vessel as a rectangular box, neglecting the ballast and sphere components as these do not contribute to the cross-sectional area in the vertical direction. The volume of this rectangular box should be equal to the volume of the gasoline tank and the air tanks added together. In our previous paper, we set the volume of the gasoline tank to be 9 times larger than the volume of the air tanks. Therefore, the total volume is simply 10 times that of the air tanks, which were calculated to be 9.93 m^3 [2]. The total volume is therefore 99.3 m^3.

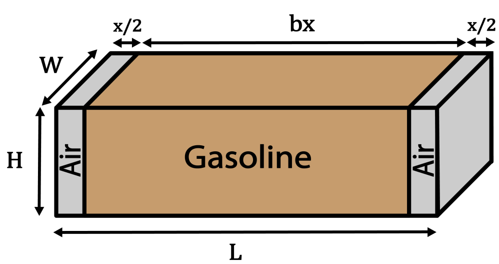

Figure 1: Volume of gasoline and air tanks.

If the width, W, and height, H, of the vessel are estimated to be ≈ 3.00m each, similar to the dimensions of a shipping container, and the volume of the air tank is 99.3 m^3, then the length can be calculated using:

$$L = \frac{V}{W \times H} \tag{10}$$

The length is therefore 11.0 m. The cross-sectional area is found by multiplying the length by the width and is found to be $A = 33.0$ m^2. The drag coefficient for a rectangular box is given by $C_D = 2.1$ [3]. The depth of the Marianas trench is $d = 10984$ m [2].

Results

The velocity during descent is calculated to be $v_{\text{sink}} = 1.47$ m s^{-1} using Eq. 7. The time taken for the descent is therefore calculated to be 2 hours and 5 minutes using Eq. 9.

The velocity of the vessel as it rises back to the surface is calculated to be $v_{\text{rise}} = 0.235$ m s^{-1} using Eq. 8. The time taken for the vessel to rise to the surface is therefore calculated to be 12 hours and 59 minutes using Eq. 9. The total travel time for a round trip from the surface to the bottom of the ocean using a Bathyscaphe is therefore 15 hours and 5 minutes.

Discussion

This paper neglects the compression of water that will take place at such depths, which will slow the descent of the vessel. Secondly, gasoline compresses twice as fast as water at such depths. As the gasoline compresses, surrounding water will fill the empty volume of the gasoline container to prevent the external pressure damaging the vessel. Water has a higher density than gasoline and so this will increase the density of the vessel and increase its rate of descent. A future paper is required to quantify the effects of compression on the travel time.

References

[1] `https://bit.ly/3lheiwv` [Accessed 30 November 2021]

[2] C. Murgatroyd, J. Stinton, C. Kinsman, and D. Mott *P2_7 Underwater Hot Air Balloon*, PST 20, (2021)

[3] `https://bit.ly/3D6g8qo` [Accessed 30 November 2021]