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P2_7 Underwater Hot Air Balloon

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Abstract

This paper analyses how a Bathyscaphe vessel, which uses gasoline to provide lift and weights and air tanks to lower the vessel, can travel to the bottom of the Mariana Trench and return to the surface again. When the air tanks are full, the vessel is neutrally buoyant on the surface with a density of $\rho_{\text{surf}} = 1022$ kg m^{-3}. When the air tanks are flooded with water, the vessel becomes negatively buoyant, with the vessel density now equal to $\rho_{\text{sink}} \approx 1100$ kg m^{-3}, thereby causing the vessel to sink. Once it reaches the bottom, the weights are dropped which causes the vessel to become positively buoyant, with the vessel density now equal to $\rho_{\text{rise}} \approx 1020$ kg m^{-3}.

Introduction

We previously proved that a hollow steel sphere with a 1m radius was capable of withstanding the pressure at the bottom of the Marianas trench. The sphere required a wall thickness of 15.7 cm and had a mass of 13.1 tonnes [1]. Transporting this sphere on a round trip from the surface to the bottom of the ocean can be done using a “bathyscaphe”[2].

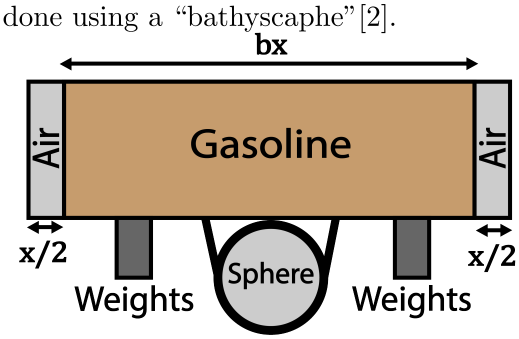

Figure 1: An image showing the different components of the bathyscaphe.

The bathyscaphe describes a steel sphere attached underneath a float. The float consists of gasoline “flotation fluid”, air tanks, and iron pellets that act as weights. Gasoline is used for flotation as its density is lower than the surrounding water and therefore provides an upwards buoyant force. The air tanks keep the vessel neutrally buoyant at the surface when they are filled with air. When the vessel plans to submerge, the air tanks are filled with water, which increases the vessel’s density, and starts the vessels descent. Finally, the weights are held in a releasable compartment so that they can be dropped once the vessel reaches the bottom. Releasing the weights will decrease the vessel’s density and allow it to return to the surface.

Equations

The overall density of the vessel is given by:

$$\rho_{\text{ves}} = \frac{M_{\text{tot}}}{V_{\text{tot}}} \qquad (1)$$

where the total mass and volumes are given by:

$$M_{\text{tot}} = \sum_{i=1}^{N} V_i \rho_i = V_1\rho_1 + V_2\rho_2 + ... + V_N\rho_N \qquad (2)$$

$$V_{\text{tot}} = \sum_{i=1}^{N} V_i = V_1 + V_2 + ... + V_N \qquad (3)$$

where ρ_i and V_i represent density and volumes of the different components of the vessel respectively. The vessel in this paper has 5 components where ρ_1 and V_1 represents the sphere, ρ_2 and V_2 the container, ρ_3 and V_3 the weights, ρ_4 V_4 the air tanks, and ρ_5 and V_5 the gasoline tank.

Values

The mass and volume of the sphere are 13100 kg and $(4/3)\pi$ m^3 respectively [1]. The mass of the container is approximated to be the same as the sphere, with a density of typical steel ≈7800 kg m^{-3}. The weights are made of iron spheres that are packed into two containers of 1 m^3 each. The density of iron is ≈ 7874 kg m^{-3}. However, the packing density of the spheres needs to be accounted for by a factor of a_o ≈0.74 [3].

	Sphere	Container	Weights
Density (kg m^{-3})	**3130**	7800	$a_o\times 7874$
Volume (m^3)	$(4/3)\pi$	**1.7**	2.0
Mass (kg)	13100	13100	**11650**

Table 1: Table of density, mass, and volume. The bold values are calculated by either multiplying or dividing the other values in the same column using $\rho = M/V$.

The densities of air and gasoline are $\rho_4 = 1.225$ kg m^{-3} and $\rho_5 = 800$ kg m^{-3} respectively. The volume of the air tanks and the gasoline tank are fixed at a 1:9 ratio, therefore $V_4 = x$ m^3 and $V_5 = 9x$ m^3. This is because the air tanks are purely to keep the density of the vessel neutrally buoyant at the surface before they are flooded with water. The volume and density values for the different components of the vessel outlined above can be substituted into Eq. (2) and (3) and the resultant terms then substituted into Eq. (1) and rearranged into:

$$\rho_{\text{ves}} = \frac{(V_1\rho_1 + V_2\rho_2 + V_3\rho_3) + x(\rho_4 + 9\rho_5)}{(V_1 + V_2 + V_3) + 10x} \quad (4)$$

Eq. (4) gives the density of the vessel as a function of the volume of the air tank, x.

Results

On the surface, the air tanks are filled with air to keep the vessel neutrally buoyant. The vessel density, ρ_{surf}, must therefore be the same as the water density ≈ 1022 kg m^{-3} [1]. Plotting Eq. (4), shown in Fig. 2, finds that the volume of the air tank must be 9.93 m^3 for $\rho_{\text{surf}} = 1022$ kg m^{-3}. Therefore, the gasoline tank is 89.4 m^3.

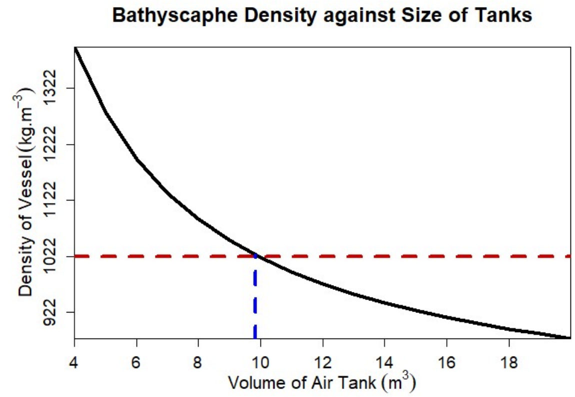

Figure 2: A graph showing the vessel density against air tank volume. Water density intercept at $x = 9.93$ m^3.

The air tanks are filled with water when the vessel decides to begin sinking, which will increase the vessel's density. The new density of the vessel whilst it is sinking, ρ_{sink}, is calculated by replacing the density of air with the density of water. Therefore, using $\rho_4 = 1022$ kg m^{-3} in Eq. (4), the sinking density is $\rho_{\text{sink}} \approx 1100$ kg m^{-3}. After reaching the bottom, the weights are released which causes the weight container to fill with water, thereby decreasing its density and causing it to rise to the surface. The density of the vessel whilst it is rising, ρ_{rise}, is calculated by replacing the density of the weights with the density of water. Therefore, using $\rho_3 = 1022$ kg m^{-3} in Eq. (4), the new density is $\rho_{\text{rise}} \approx 1020$ kg m^{-3}. This allows the vessel to rise to the surface and complete its round trip.

References

[1] C. Murgatroyd, C. Kinsman, J. Stinton, and D. Mott, *P2_6 SubMariana*, PST 20, (2021)

[2] `https://bit.ly/3lheiwv` [Accessed 30 November 2021]

[3] `https://bit.ly/3D06ohe` [Accessed 30 November 2021]