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A5_7 Warming Up Your Hands

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Abstract

This paper explores how effective rubbing your hands together is to generate heat, and extrapolates to find the upper limits of what can be done whilst neglecting heat loss to the surroundings. We find that for a small applied force, the temperature gain from the act of rubbing one hand over the other is $T = 9.39 \times 10^{-4} K$, whereas for the strongest push force for a human arm the temperature gain is $\delta T = 0.085K$. For the smaller temperature gain, it is shown to take 5.79 hours to form first degree burns, whereas for the high temperature gain it can take as little as 3.8 minutes.

Introduction

On cold winter days, it isn't uncommon to see people rubbing their hands together to warm them. This heating is caused by the friction between the two hands. The effectiveness of this heating is dependant on many factors, and in some cases may be overwhelmed by radiative cooling. Therefore, by neglecting cooling by the surroundings we can estimate the temperature gain each time one hand is rubbed over the other, thus determining if this is an effective mechanism for warming of the hands.

Heating Equations

To begin deriving a formula for the effectiveness of a single rotation of rubbing one hand over the other, we start with the equation for the heat flux across a single hand. This will be done by assuming that the hand surface is completely flat and uniform. The equation for heat flux generated by two semi-infinite surfaces rubbing against each other is given by:

$$q = \frac{\sigma \mu v}{2} \tag{1}$$

where q is the heat flux put into each surface, σ is the normal stress given by $\sigma = \frac{F}{A}$, and μ is the coefficient of friction for the surface and v is the velocity of one surface relative to the other. To find the temperature increase for a specific material, we use the following equation:

$$\Delta T = \frac{q}{K}\sqrt{\frac{4\alpha t}{\pi}} \tag{2}$$

where ΔT is the temperature increase, K is the thermal conductivity, α is the thermal diffusivity and t is the time. By combining Eq. (1) and Eq. (2), we can obtain an equation for the temperature increase for each time one hand is slid over the other, using $\alpha = \frac{K}{\rho C_p}$.

$$\delta T = \sigma \mu v \sqrt{\frac{t}{\pi K \rho C_p}} \tag{3}$$

where δT represents the increase in temperature per rotation (rather than the total temperature increase denoted by ΔT), ρ is the density of the material and C_p is the specific heat capacity for the material. For finite objects, this relation is only valid for $t \ll \frac{L^2}{\alpha}$, where L is the characteristic length of the hand, meaning the time for one rotation must be small compared to the size of the object.

Heating for Skin

R. Webb et al [1] give good estimates for the thermal properties of skin found across the body. In this case, we will be looking at the properties of skin on the palm of the hand. For the skin on a human palm, we find the following properties:

α	$0.15mm^2s^{-1}$
ρC_p	$2.4jcm^{-3}K^{-1}$
K	$0.36Wm^{-1}K^{-1}$
μ	0.62 ± 0.22

Table 1: A table containing all of the parameters for human palm skin required for this equation, taken from Webb et al (2015).

If we assume it takes $0.2s$ to slide one hand over the other, with the average human male hand length being $19.49cm$ [2], we can find a velocity of approximately $0.5ms^{-1}$. The average surface area of a human hand is $0.8448m^2$ so if we assume a force applied to the system of $5N$ we find a normal stress, $\sigma = 11.16Nm^{-2}$. If we put all of these parameters into Eq. (3), we find a temperature increase per rotation of $\delta T = 9.39 \times 10^{-4}K$. This is a very low temperature increase per rotation, making it very inefficient.
If we look at Eq. (3), we can see that the increase in temperature per rotation is directly proportional to the velocity of the hands, the normal stress, and $t^{1/2}$. Therefore, to increase the temperature gain per rotation, the easiest option is to increase the force applied. By using the maximum push force of a human male taken from B. Das and Y. Wang (2004) [3], we have a push force per arm of $227N$, meaning a new normal stress of $10133.92Nm^{-2}$. This gives us a temperature increase per rotation of $\delta T = 0.085K$. This is much more meaningful, as it allows for a greater temperature increase per rotation, although it may be hard to continually apply this force.

In order to obtain first degree burns, skin must reach a temperature of $118°C$, with second and third degree burns occuring at $131°C$ and $162°C$, respectively [4]. Assuming the skin is at room temperature $T_0 = 20°C$, for the lower estimate of δT, it would take 104366.3 rotations to cause first degree burns, with second and third degree burns requiring 118210.8 and 151224.7 rotations each. Assuming a time per rotation of 0.2 seconds, this would take 5.79 hours for first degree burns, 6.5 hours for second degree burns and 8.4 hours for third degree burns.
For the high-force rubbing, we obtain much smaller numbers, with first degree burns requiring 1153 rotations, and second and third degree burns requiring 1306 and 1671 rotations each. This means you would need to rub your hands together for 3.8 minutes for first degree burns, 4.3 minutes for second degree burns and 5.6 minutes for third degree burns.

Conclusion

When hands are rubbed together with a low normal force, not much heat is generated, so when heat losses to the surroundings are considered, the temperature gain is negligible compared to the temperature variations of the surroundings. Therefore, to get the best temperature change a strong force must be applied to increase the value of δT. When using a high normal force, the temperature increase from friction can begin to become noticeable and will increase the temperature of the surface of the palms faster than the heat can be dissipated by the surroundings. In a lot of these cases however, it is unreasonable to assume someone would rub their hands together for long enough to burn the skin, as the surface would most likely be rubbed smooth by this point causing extreme pain. These equations do however give us some important information on the most effective ways to warm your hands. By increasing the friction of the surfaces, the normal stress on the surfaces or the velocity of the movement the temperature can be increased at different rates leading to shorter bursts of more impactful heating.

References

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