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A2_5 Go Long, Earthlings!

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Abstract

This article investigates whether an American football shaped planet such as KOI 1843.03 could exist in our Solar System and whether or not it could be potentially habitable. It was found that the smallest orbital radius at which such an Earth-mass planet could occur is $a = 8.26 \times 10^8\ m$, if it is made of pure iron, but that the effective surface temperature (assuming an Earth-like albedo value of $\alpha = 0.3$) would be $T_e = 3428\ K$ (or $3155°C$) which far exceeds the habitable limit.

Introduction

The Earth is not flat. This can be observed and proven theoretically. To start with, the gravitational force of attraction between two objects scales inversely with the radial distance between them and proportionally to their respective masses, as shown by:

$$F_{grav} = \frac{Gm_1 m_2}{r^2} \tag{1}$$

where G is the gravitational constant, m_1 and m_2 refer to the masses of two respective objects and r is the spatial separation between the objects' centres of mass. But the Earth is not completely spherical either. The Earth's rotation causes oblation of the Earth around the equator, resulting in an equatorial bulge whereby the equatorial radius of Earth is approximately 43 km larger than the pole-to-pole radius [1]. All rotating bodies experience this effect, with the equatorial bulge size scaling with rotation speed. As such, one can imagine that extremely fast rotating bodies might begin to form irregular shapes.

We know that low-mass bodies can form irregular shapes without high rotation speeds (e.g. Gaspra, Ida and Mathidle as discovered by the Galileo spacecraft [2]) but what extreme shapes could a more massive, planet sized object achieve?

A recent paper [3] has discovered the transiting planet candidate KOI 1843.03 with the shortest known orbital period of any planet. It is believed that the planet is so stretched by its fast orbital speed that it is shaped like an "American football" with an aspect ratio (the ratio of the maximum length to the minimum width) of up to 1.79.

As such, this article will investigate the question - is it possible for an Earth-like planet to form this shape in our own Solar System and would life be able to survive on its surface?

Theory

The extreme elongation of KOI 1843.03 is due to strong tidal forces on the planet due to its close proximity to its host star, rather than the rotational effects seen on Earth. Close, fast orbiting planets become elongated along the planet-star axis [4] assuming that the planet has a strong enough composition to resist being torn apart. The minimum distance at which a planet made of an incompressible fluid can avoid being torn apart is defined by the Roche limit [5],

$$a_{limit} = 2.44R_* \left(\frac{\rho_*}{\rho_p}\right)^{\frac{1}{3}} \quad (2)$$

where a is the minimum semi-major axis of the planet, R_* is the radius of the star and ρ_* and ρ_p are the densities of the star and planet respectively.

The paper [3] determined the Roche limit for various planets with varying compositions to make predictions of KOI 1843.03's composition, but also determined that the limiting orbital period of a pure iron, Earth mass planet is 3.6 hours. We also follow this paper's approach of ignoring relativistic effects on the shape of the planet and also assume a perfectly spherical orbit. In reality only 32% of Earth's mass is due to its iron-dominated core [6] but to investigate the limiting conditions in our own Solar System we will assess the maximum 100% iron case.

Results and Discussion

By applying the results from paper [3], the semi-major axis of this pure iron, Earth-mass planet with an orbital period of 3.6 hours can be found using Kepler's 3^{rd} law,

$$a = \left(\frac{T^2 G M_*}{4\pi^2}\right)^{\frac{1}{3}} \quad (3)$$

where T is the orbital period of the planet, G is the gravitational constant and M_* is the mass of the star (in this case the Sun). Substituting in the values gives $a = 8.26 \times 10^8\ m$ which is larger than the Sun's radius of $6.96 \times 10^8\ m$ and so it could exist. However, one major component for assessing a planet's suitability for life is the surface temperature of the planet as this assesses whether liquid water can exist on its surface (the condition set to determine the possible habitability of a planet). A planet's effective surface temperature can be calculated using,

$$T_e = \left(\frac{(1-\alpha)S}{4\sigma}\right)^{\frac{1}{4}} \quad (4)$$

where α is albedo (with $\alpha_{Earth} = 0.3$ [7] on average) σ is the Stefan Boltzmann constant, and S is radiative flux from the star at the planet's orbital radius. To calculate S, we can use,

$$S = \frac{Solar flux}{Area_{sphere}} = \frac{3.84 \times 10^{26}}{4\pi d^2}. \quad (5)$$

Substituting in the semi-major axis for d, we find that $S = 4.474 \times 10^7\ Wm^{-2}$. Substituting this back into Eq. (4), we find that the effective surface temperature of this planet would be $T_e = 3428\ K$ (or $3155°C$).

Conclusion

The effective surface temperature found for a pure-iron planet orbiting at the closest possible distance to our Sun before tidal forces tore it apart was found to be 3428 K. This is 4.78 times hotter than the melting point of aluminium, and is also almost hot enough to melt carbon itself (carbon's melting point is 3500°C). As such, it would definitely not be suitable for hosting life as we know it on its surface.

References

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