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P3_8 Tunnel to the centre of the Earth

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Abstract

If a tunnel was dug between the North and South pole, then a person, modelled as a particle would fall to the centre of Earth at a terminal velocity of 20,000 m/s. It would take around 300 s for the particle to reach the centre of the Earth. At the centre, the particle would oscillate in simple harmonic motion unless the tunnel was a vacuum and the person would never reach the other side.

Introduction

Imagine you dig a tunnel from the North pole to the South pole, along the rotational axis, and then jump into that tunnel. This paper aims to work out the terminal velocity the person would fall at; how long it would take to reach the centre of the Earth; and what forces the person would experience due to gravity.

Assumptions

In order to tackle this problem, first we must make a number of assumptions. Firstly the person will be modelled as a spherical particle. Pressure, radiation, and heat effects will not affect the particle (it is shielded in a protective suit). Any rotational affects can be ignored as the tunnel is dug from the North to the South pole. The walls of the tunnel are strong enough to prevent any collapsing. We make the assumption that the density of Earth is constant.

Theory

As the particle is falling through the tunnel, it will accelerate until it reaches terminal velocity. This is when the resistive forces become equal to the weight of the object due to gravity and the resultant force on the particle is zero [1]. Using Eq.(1) the terminal velocity, V_t of the particle can be worked out.

$$V_t = \sqrt{\frac{2mg}{\rho A C_d}} \tag{1}$$

where m is the mass of the particle, g is the acceleration due to gravity, ρ is the density of Earth, A is the projected area of the particle, and C_d is the drag coefficient. In the scenario above we take the mass of the particle to be the mass of an average human at 62 kg [2]. Since the person is modelled as a spherical particle, the drag coefficient is taken for a sphere and is 0.5 [3]. The density of Earth is constant at 5520 kg/m^3 [4]. The projected area is set at 1 nm^2.

From the terminal velocity we can work out how long the particle would take to fall to the centre of the Earth, using Eq.(2).

$$t = \frac{s}{V_t} \tag{2}$$

where t is the time taken and s is the distance the particle has travelled, which to reach the centre of Earth would be 6371 km [5].

Now consider the force of gravity acting on the particle, using Gauss' Law of Gravity, Eq.(3).

This law is used by first imagining a closed sphere of radius x which is drawn from the centre of the Earth.

$$F = \frac{-Gm_p m_e}{x^2} \tag{3}$$

where G is the gravitational constant, m_p is the mass of the particle and m_e is the mass enclosed by the sphere. The mass enclosed by the sphere will decrease as the particle falls since the radius, x, decreases and hence the force will also.

In order to find m_e, we first must obtain an equation for the mass of the Earth, M_E, Eq.(4) and then sub this into the equation for m_e, Eq.(5), which gives Eq.(6).

$$M_E = \frac{4}{3}\pi R^3 \rho \tag{4}$$

where R is the radius of the Earth, 6371 km and ρ is the density of the Earth.

$$m_e = \frac{4}{3}\pi x^3 \rho \tag{5}$$

$$m_e = M_E \frac{x^3}{R^3} \tag{6}$$

Substitute Eq.(3) and Eq.(6) into Eq.(7) which gives Eq.(8),

$$F = m_p a \tag{7}$$

where a is the acceleration.

$$a = \frac{-Gm_e}{R^3} x \tag{8}$$

Since acceleration can be expressed using equation Eq.(9), we can determine that this is the equation for simple harmonic motion.

$$a = \frac{d^2 y}{dt^2} \tag{9}$$

Hence the particle will fall to the centre of Earth and then oscillate at simple harmonic motion, given by Eq.(10).

$$\omega^2 = \frac{-Gm_e}{R^3} \tag{10}$$

Results

The terminal velocity for the particle falling through the tunnel is found to be 20,000 m/s and it would take around 300 s for the particle to fall to the centre of the Earth. At the centre the particle would oscillate in simple harmonic motion with an amplitude of R and a frequency given by ω.

Conclusion

As the particle is falling further down the tunnel, the radius of the enclosed sphere gets smaller and the net gravity pulling the particle down decreases. When it reaches the centre of Earth, there will be no net gravity acting on the particle. Inertia will carry the particle to just past the centre of the Earth, the net gravity would slow until the particle started to fall back through the centre. This is what causes the particle to oscillate back and forth in simple harmonic motion.

References

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