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# P5_4 Bunker Busters: A Gift from Space 

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#### Abstract

The effectiveness of a 10 m Tungsten rod, dropped from LEO, is evaluated as a practical bunker buster and was found to impact at the surface at Mach 23, producing a non-explosive yield $\approx 417$ times that of the GBU-57 Massive Ordnance Penetrator (MOP), while having a similar penetration through soil and a maximum depth of a 22 m through solid steel.


## Introduction

An intercontinental ballistic missile has been found to be cheaper and significantly more destructive than hypothetical orbital bombardment methods. However, a kinetic rod as a hyper-sonic streamlined projectile could have more locally devastating effects on specific underground structures than existing subsonic "bunker busters".

## Model

The rod is modelled as a parabolic cone connected to a long cylinder, shown by Eq.(1).

$$
\begin{equation*}
V=V_{c y l}+V_{\text {cone }}=\frac{\pi d^{2} l}{4}+\frac{2 \pi d^{2} h}{15} \tag{1}
\end{equation*}
$$

Hence where $d$ is the diameter of the rod, $h$ is the height of cone and $l$ is the length of the rod, the mass of the rod, $m$ is given by Eq.(2).

$$
\begin{equation*}
m=\rho_{W} V=\pi d^{2}\left(\frac{2 h}{15}+\frac{l}{4}\right) \rho_{W} \tag{2}
\end{equation*}
$$

Atmospheric re-entry temperature is mainly dependent on surface area and speed, thus $d$ is minimised. The fastest re-entry speed of any synthetic object was reached by the spacecraft
"Stardust", which achieved temperatures of $\approx$ 3200 K [1]. Tungsten, with a density of $\rho_{W}=$ $19300 \mathrm{~kg} \cdot \mathrm{~m}^{-3}$, has the highest melting point of all pure metals at 3680 K [2], making it perfect for withstanding high levels of re-entry heating.

## Impact Energy

Considering the satellite which launches the kinetic rod, is in circular orbit at an altitude of $a=400 \mathrm{~km}$, the tangential speed of satellite is given in Eq.(3), where $r=R_{\oplus}+a$.

$$
\begin{equation*}
v_{t}=\sqrt{\frac{G M_{\oplus}}{r}} \tag{3}
\end{equation*}
$$

An orbital transfer is made from low earth orbit (LEO) to ground, $\Delta r$, minimising the angle of inclination, so the rod is vertical at low altitudes, where $r_{0}=R_{\oplus}$ and the de-orbit burn is $\Delta v_{t}$.

$$
\begin{gathered}
d v_{t}=-\frac{\sqrt{G M_{\oplus}}}{2} r^{\frac{-3}{2}} d r \rightarrow \Delta v_{t}=-\frac{v_{t}}{2 r_{0}} \Delta r \\
\therefore v_{d r o p}=v_{t}+\Delta v_{t}
\end{gathered}
$$

The drop velocity of the kinetic rod considers the tangential velocity minus the de-orbit burn, which leads to the velocity of impact found in

Eq.(4) as a result of the additional acceleration due gravity.

$$
\begin{equation*}
v_{i}=\sqrt{v_{d r o p}^{2}+2 g \Delta r} \tag{4}
\end{equation*}
$$

The kinetic energy associated with the rod at the moment of impact is given by Eq.(5).

$$
\begin{equation*}
K_{i}=\frac{1}{2} m v_{i}^{2} \tag{5}
\end{equation*}
$$

## Penetration

The extremely high pressures upon impact allow for a hydrodynamic penetration model to be used, where the rod's momentum will continue into the ground as it penetrates, until it's completely eroded due to the structural failures causing mass loss through melting. The use of Bernoulli's equation finds the the root density law of high inertia impacts for a max penetration length, $L_{m}$, against a ground density, $\rho_{g}$, for a total rod length, $L=l+h$, given by Eq.(6).

$$
\begin{equation*}
L_{m}=\sqrt{\frac{2 \rho_{W}}{\rho_{g}}} L \tag{6}
\end{equation*}
$$

## Results

The specifications proposed for "Project Thor" inspired the following dimensions for the rod, in which $d=1 \mathrm{~m}, l=8 \mathrm{~m}, h=2 \mathrm{~m}$, producing a rod mass, $m=137.43 \times 10^{3} \mathrm{~kg}$.

Given the initial height at LEO, the drop velocity is found $v_{\text {drop }}=7431.5 \mathrm{~m} \cdot \mathrm{~s}^{-1}$ and hence the velocity of impact, $v_{i}=7942.0 \mathrm{~m} \cdot \mathrm{~s}^{-1}$. Thus the associated kinetic energy upon impact, $K_{i}=$ $4.33 \times 10^{12} \mathrm{~J}$, equates to $\approx 1 \mathrm{kT}$ of TNT.

| Substance | Density <br> $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | Penetration <br> $(\mathrm{m})$ |
| :---: | :---: | :---: |
| Dry soil (clay) | $1600[3]$ | 49.12 |
| Concrete (dense) | $2400[4]$ | 40.10 |
| Steel | $7820[4]$ | 22.22 |

Table 1: Maximum penetration of 10 m rod at different ground surfaces, using Eq.(6)

## Discussion and Conclusion

Upon impact, the eroding rod transfers most of its kinetic energy into the ground which is equal to the explosive yield of $\approx 417$ GBU- 57

MOPs [5] (the US' leading bunker buster). With a penetration twice its length through steel and 5 times its length through dry soil, from Table (1), the rod is just under the 61 m penetration of the GBU-57 [6]. An increase in $L$ would result in proportionally higher penetrations where at $L$ $=20 \mathrm{~m}$, the penetrations through soil would be $61 \%$ higher than the GBU- 57 .

A de-orbit manoeuvre could be made using a thruster which would detach once burn is achieved, while fins would stabilise the descent of the rod. As the derivation of including drag, with varying pressure-height, was too complex to fit into this article, it was neglected along with radially dependent $g$, which was kept constant. As it was only significant at low altitudes, a model including drag would drastically decrease the impact velocity by about $1000 \mathrm{~km} \cdot \mathrm{~s}^{-1}$, so the impact energy would be around $25 \%$ lower, but since the rod would still retain a hyper-sonic velocity of Mach 20, the impact would remain hydrodynamic and penetration lengths would stay the same.

## References

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