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A6_5 A Bright Halo

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Abstract

In the video game series Halo, there are planet sized ring megastructures known as Halos. In these structures, the inside of the ring looks like Earths terrain. Approximately half of the inner ring is illuminated by its local star and the other side is not. The Halo then rotates to simulate night and day. In this paper, to determine the relative brightness at midnight, we obtain an approximate value for the power per unit area received from light reflected off the lit up parts of the Halo at midnight. This value is 2.1 W/m^2, which is 0.16% of the power received on Earth at normal incidence from the Sun.

Introduction

In the Halo Universe, Halos are massive ring shaped structures about the size of Earth. About half of the inner ring is lit up by its local star and the other half is in shadow. The part in shadow of the Halo will receive light reflected from the lit up parts of the Halo. In this paper, we calculate the total flux received at the midnight part of the Halo from this effect.

Theory

The star is slightly out of the plane of the Halo and is very far away. Therefore, the light rays run parallel to each other and we can treat the rays as if the angle between the plane of the Halo and the star is zero. We will consider light rays that hit the other side of the Halo and are reflected to the point on the Halo at which it is midnight. With reference to Figure 1 below, this point is point A. The angle between the ground and the light ray at A is ϕ. At an arbitrary point in the circle that is lit up B, the ground is at an angle θ to the light ray. This means that the angle between the centre of the circle at O and the horizontal is θ. We can then integrate the reflected flux contributions for θ between 0 to π, that is the star lit area, to find the total contribution. We also let the radius of the Halo be a and the distance from B to A be r.

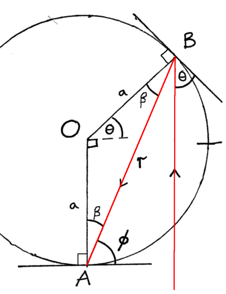

Figure 1: Diagram showing a light ray represented by the red lines reflecting off the Halo at B and hitting point A. The angle β is $\pi/2 - \phi$.

From Figure 1, it can be seen that the lines OA, AB, and BO form an isosceles triangle. Hence,

$$\phi = \frac{\theta}{2} + \frac{\pi}{4}. \tag{1}$$

If the flux from the star is F, then the irradiance E_e at point B is:

$$E_e = F \sin\theta. \tag{2}$$

The albedo α of a surface is defined as the ratio of the radiosity J_e to the irradiance,

$$\alpha = \frac{J_e}{E_e}. \tag{3}$$

Using Eq. (3) and Eq. (2) gives J_e as $J_e = \alpha F \sin\theta$. Hence, the total power dP given out by an infinitesimal area dA is $dP = J_e dA = \alpha F \sin\theta dA$. The area dA is given by $aLd\theta$, where L is the width of the Halo. Hence, $dP = \alpha F \sin\theta aLd\theta$. The solid angle Ω subtended by the infinitesimal area dA' at A is:

$$\Omega = \frac{dA'}{r^2} \sin\phi. \tag{4}$$

Hence, the power dP' received by the area dA' is:

$$dP' = \frac{\Omega}{2\pi} dP = dA' \frac{\alpha F a L}{2\pi r^2} \sin\phi \sin\theta d\theta, \tag{5}$$

where the 2π comes from the fact that the power is reflected equally throughout a hemisphere and hence has solid angle 2π. Hence, the power per unit area is:

$$\frac{dP'}{dA'} = \frac{\alpha F a L}{2\pi r^2} \sin\phi \sin\theta d\theta. \tag{6}$$

Using the cosine rule,

$$r^2 = 2a^2 - 2a^2 \cos(\theta + \frac{\pi}{2}) = 2a^2(1 + \sin\theta). \tag{7}$$

Substituting this into Eq. (6) gives:

$$\frac{dP'}{dA'} = \frac{\alpha F L}{4\pi a} \frac{\sin(\frac{\theta}{2} + \frac{\pi}{4}) \sin\theta}{(1 + \sin\theta)} d\theta, \tag{8}$$

where we have used Eq. (1) to substitute out ϕ. Integrating from the dawn to dusk sections of the ring tells us that the total power per unit area received at A F_A is:

$$F_A = \frac{\alpha F L}{4\pi a} \int_0^{\pi} \frac{\sin(\frac{\theta}{2} + \frac{\pi}{4}) \sin\theta}{(1 + \sin\theta)} d\theta. \tag{9}$$

This integral evaluates to 1.06568. Hence, F_A is given by:

$$F_A = 1.06568 \frac{\alpha F L}{4\pi a}. \tag{10}$$

Now that we have this formula for the power received per unit area at A, all that is left to do is substitute in the values for this specific situation. Assuming that the Halo has the same albedo as Earth, the value of α is 0.29 [1]. If the Halo has a similar star to Earth and is at a similar distance from the star as the Earth is from the Sun, then the value of F is about 1360 W/m^2 [2]. The radius of the modern Halo structure is 5,000 km [3]. So, $a = 5 \times 10^6 m$. The surface of the Halo ring is 318 km wide [3]. Hence, $L = 318,000m$. Putting these values into Eq. (10) gives the value for the total power per unit area reaching A as 2.1 W/m^2.

Discussion and Results

The value obtained for the power per unit area received at midnight on the Halo was $2.1W/m^2$. Comparing this to the total solar irradiance of $1360W/m^2$, it is 0.16% of the total solar irradiance. However, a number of assumptions have been made. We have assumed that the light rays are only reflected once. In reality, the light rays could be reflected more than once and still reach A. This would increase the Power per unit area received at A. We also did not consider the fact that the angle between the plane of the Halo and the star would decrease the power received from the star. Finally, we assumed that half of the Halo would be lit up.

References

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[2] Kopp, G. and Lean, J.L., 2011. *A new, lower value of total solar irradiance: Evidence and climate significance*. Geophysical Research Letters, 38(1).

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