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A6_3 Travelling to the Future

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Abstract

In the special theory of relativity, a moving clock runs slow. This means that if a person was to move at high speeds away from Earth and then eventually return, they would have aged less than someone stationary on the Earth. In this paper, we determine the maximum theoretical amount of time the average person could travel to the future using this method and the velocity of the person as a function of the person's proper time as they take the trip. We found that the furthest a person could travel to the future is 267 million years. This answer depends on both the acceleration of the rocket as measured by the person on the rocket and how long the trip takes as measured by the person on the rocket. However this value of 267 million years is highly sensitive, with testing in the paper showing that with a delta of just 10% to either the trip time in the person's frame or to the acceleration can change the answer magnitude from hundreds of millions to billions of years

Introduction

We consider a hypothetical scenario in which a person who has an average life span is sent off in a rocketship at birth. After a long period of acceleration, its acceleration changes direction back towards Earth. Then it changes direction again in order to ensure that it is at rest with respect to Earth when it arrives. From Earth's perspective, the person's proper time (which is the time as measured from the person's perspective) runs slow. We determine how far into the future the Earth will be when the person arrives back on Earth. We take the time of the trip to last the lifetime of the person when measured by that person's own clock.

Method

In order to travel the farthest distance into the future, the magnitude of the rockets acceleration a, must be the largest constant value survivable by a human for prolonged periods of time. Taking the time of the total trip as measured by the clock on the rocketship to be T, the rocket must accelerate at a rate of a as measured from the rockets frame for arbitrary proper time τ between 0 and $T/4$, -a for τ between $T/4$ and $3T/4$ in order for the rocket to turn around, and finally a for τ between $3T/4$ and T in order for the rocket to be stationary with respect to Earth as it arrives back on Earth. Therefore the acceleration of the rocketship as measured on the rocket as a fuction of proper time $a'(\tau)$ is given by,

$$a'(\tau) = \begin{cases} a, & 0 \leq \tau < \frac{T}{4} \\ -a, & \frac{T}{4} \leq \tau \leq \frac{3T}{4} \\ a, & \frac{3T}{4} < \tau \leq T \end{cases} . \tag{1}$$

The equation relating the acceleration of the rocketship, as measured on the Earth du/dt, and the acceleration as measured on the rocketship is

[1],

$$\frac{du}{dt} = \frac{1}{\gamma^3} a'(\tau), \tag{2}$$

where,

$$\gamma = \frac{dt}{d\tau} = \frac{1}{\sqrt{1 - u^2/c^2}} \tag{3}$$

is the Lorentz factor and c is the speed of light. Substituting the mathematical relationship,

$$\frac{du}{dt} = \frac{d\tau}{dt}\frac{du}{d\tau} = \frac{1}{\gamma}\frac{du}{d\tau} \tag{4}$$

into Eq. (2) and rearranging gives,

$$\int \frac{du}{1 - \frac{u^2}{c^2}} = \int a'(\tau)\, d\tau. \tag{5}$$

After integration, we find that the velocity of the rocketship as measured on Earth as a function of proper time is given by,

$$u(\tau) = \begin{cases} c \tanh \frac{a}{c}\tau, & 0 \le \tau < \frac{T}{4} \\ c \tanh \frac{a}{c}(\frac{T}{2} - \tau), & \frac{T}{4} \le \tau \le \frac{3T}{4} \\ c \tanh \frac{a}{c}(\tau - T), & \frac{3T}{4} < \tau \le T \end{cases}. \tag{6}$$

Using this expression for u in equation Eq. (3) and integrating over the total proper time gives the total time it takes on Earth as,

$$t = \int_0^T \frac{1}{\sqrt{1 - \frac{u(\tau)^2}{c^2}}}\, d\tau. \tag{7}$$

Which evaluates to,

$$t = \frac{4c}{a} \sinh \frac{Ta}{4c}. \tag{8}$$

This equation can now be used to estimate how long the average human could travel to the future using this method. The average human life span today is 72.6 years[2]. This means that the value of T can be taken as 72.6 years. If we assume that the maximum acceleration survivable by humans for extended periods of time is the same as the acceleration on Earth g, then the value of t obtained from Eq. (8) is 267 million years. Here, the value of g was taken to be $9.81 ms^{-2}$ and the value of c was taken to be $2.998 \times 10^8 ms^{-1}$.

Discussion

In order to test how sensitive the answer of 267 million years is to the acceleration and the lifetime of the person, we can choose to increase T and a by an arbitrary amount to see how much the value of t increases. Putting the appropriate values into Eq. (8) shows that t increases by a factor of about 6.5 when T is increased by 10% and is increased by a factor of 5.9 when a is increased by 10%.

Conclusion

Our conclusion is that the furthest a person who lives an average lifespan could travel to the future is 267 millions years if they travel at the comfortable acceleration of $9.81 ms^{-2}$. However, the time travelled to the future is very sensitive to the acceleration and to the lifetime of the person. An increase of just 10% to either of them will send the answer into billions of years, not just hundreds of millions of years. If they arrive back after billions of years, the Earth will have become inhospitable for humans[3] and unfortunately, they won't be able to go back.

References

[1] d,Inverno, R 1992, *Introducing Einstein's relativity*, Oxford University Press, Oxford, p. 37.

[2] `https://ourworldindata.org/life-expectancy` [Accessed 23 October 2021]

[3] Franck, S., Block, A., von Bloh, W., Bounama, C., Schellnhuber, H.J. and Svirezhev, Y., 2000. Reduction of biosphere life span as a consequence of geodynamics. Tellus B, **52**(1), pp.94-107