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A3_2 Diverging Droplet

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Abstract

Droplets of a fluid confined between two plates are modelled as a diverging lens. A lens construction method is devised where the radii of curvature at each interface are related to the desired lens power and the plate separation. However, this lens system is found to be flawed by a limited field of view and distorted image due to gravitational effects and the 3D droplet curvature.

Introduction

Droplets of a fluid trapped between two planes are subject to surface tension that generates an adhering force proportional to the pressure difference at the fluid-air interface [1]. If the plate surfaces are ”wetting” [2] a diverging lens is formed, Figure 1; generated by adhesive and cohesive forces.

To form the lens, the fluid should be in contact with both plates and in equilibrium for stability over long time-scales.

Thus, the aim of this investigation is to explore the potential and practical feasibility of such a diverging lens created with water.

Discussion

The lens model of the confined fluid droplet between two plates is indicated in Figure 1, where h is the parallel plate separation and r_1 and r_2 are the radii of curvature at each interface.

The air-fluid interface is approximately spherical by assuming gravitational effects and pressure variation with height at these small scales is trivial.

The adhesion force, F, at the fluid-plane interface is expressed in the form:

$$F = \frac{2\gamma V \cos\theta}{h^2} \tag{1}$$

Where the variables γ, V, θ and h represent the surface tension, fluid volume, contact angle and the parallel plate separation, respectively.

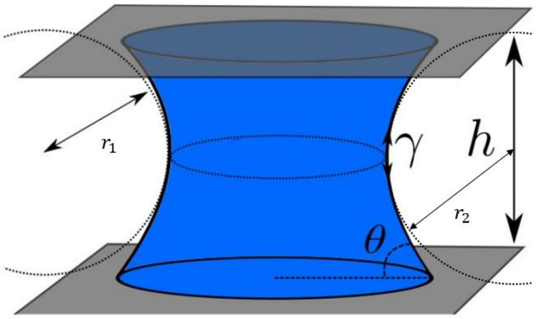

Figure 1: Modified diagram from [1]; constricted fluid between two plates. r_1 is equal to r_2 if the plates are parallel, i.e. h is constant.

To demonstrate how a desired lens can be produced using this model we aim for the magnification of the lens to be that prescribed for moderate myopia (short-sightedness). Thus, the required droplet lens properties are evaluated by considering a 2D cross-section of the lens and applying the lens-maker's equation.

The lens-maker's equation, Eq (2), describes a lens with focal length, f, relative refractive index of the lens, n, and radii of curvature at each interface, r_1 and r_2, respectively.

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \tag{2}$$

The average power of a lens, $(\frac{1}{f})$, for moderate myopia is -3.00 dioptres [3], corresponding to focal length of $-\frac{1}{3}$m. Thus, utilising Eq (2) and the relative refractive index between water and air, $n=1.33$ [4], the subtraction of the reciprocals of r_1 and r_2 individually is equal to ~ -9.09.

Thus, Eq (2) demonstrates that the plates can be parallel or at a relative angle, and that any combination of r_1 and r_2 can be used which satisfies -9.09, the subtraction of the individual reciprocals of r_1 and r_2. When evaluating two radii values, the sign convention for the lens-makers equation must be adhered to.

To control the radii of curvature at the two fluid-air interfaces the plate separations at these interfaces, which may not be equal, must be controlled with accuracy during lens construction.

The geometry in Figure 1 indicates the relationship, Eq (3), between the plate separation $h_{1,2}$ corresponding to the radius of curvature, $r_{1,2}$, where $\theta_{1,2}$ can be experimentally determined.

$$h_{1,2} = 2r_{1,2}cos\theta_{1,2} \tag{3}$$

If a desired lens requires a large curvature, Eq(3) indicates a larger plate separation at the given interface. However, this decreases the adhesive force significantly due to their inverse square relationship indicated by Eq (1) and could lead to detachment of the fluid from one of the plates. Consequently, a larger droplet volume to increase the adhesive force at both fluid-plate interfaces can be used, but this would also increase the gravitational force on the droplet by the same factor. Gravity is expected to deform the curvature of the fluid, hindering the relationship of Eq (3) and diverging from Eq (1), which assumed neglected gravitational forces. Quantifying this deformation goes beyond this paper.

Additionally, since the volume and adhesive force are related linearly as opposed to a square relation, a significantly larger volume would be required to offset the effect of an increased plate separation.

Curvature of the droplet in 3D would also deform the image seen by the observer due to varying incident and refracted angles. This may be overcome by a higher circular cross-section of the droplet projected on the plate by keeping the plate separation low. Gravity effects can also be minimised using this solution, but the field of view becomes limited and vertical disturbances can still effect the droplet diameter [2].

Conclusion

A lens model was proposed, consisting of a fluid droplet confined by two plates. Potential of achieving various lens powers was explored in relation to the physical parameters involved. The system was limited by requirement of larger fluid volumes, which becomes hard to practically control and shield from external disturbances such as gravity and vibrations. Thus, a limited field of view, distorted image and possible total internal reflection makes the lens less practical.

References

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