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A2_2 Ignoring Air Resistance

Mac McMullan, Tom Filip, Megan Perks, Natalia Stylianou

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

Often in PST journal articles, "air resistance" in outer space is disregarded. We establish a model of how a "frictional" air resistance force would work in the cold, diffuse environment of a planetary nebula, and in which cases it became significant, which was found to be close to the speed of light.

Introduction

Space is not actually a vacuum, but in most cases we can approximate it to be one. However, if an object is especially large or fast, the force on the object caused by collisions with the nebula material will cause it to decelerate. For the purposes of this paper, we will assume that the object in question (a spaceship) is able to deflect all objects it collides with elastically, without deforming.

Theory

For our analysis, we assume that we have an object with a collision area A in the direction of travel. We can assume that the particles within the nebula are, overall, stationary within their own frame of reference, and there would be no velocity preferentially in one direction or another.

If we assume that the object is travelling at a constant velocity v, and we imagine ourselves within the reference frame of the object, then the particles of the nebula are approaching the ship with a speed equal to $-v$ (Figure 1). We can assume all particles are of the same mass m, for which we will eventually use the mass of ionised hydrogen, given that this is what the vast majority of the material of a dense planetary nebula is made of [1]. We can define the volume of space swept per unit time to be:

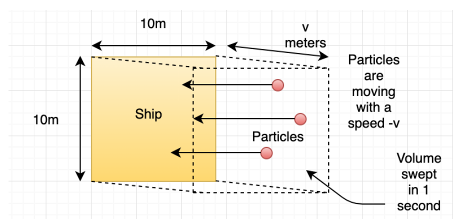

Figure 1: This is a diagram of the model for calculating air resistance.

$$\Delta V = \frac{Av}{\Delta t}, \tag{1}$$

where Δt is an arbitrary unit of time, measured in seconds. The total number of particles in a given volume, N, is given by:

$$N = nV, \tag{2}$$

where n is the number density of particles within one unit volume. In Tipler [2], momentum is given as:

$$p = mv \tag{3}$$

The equation for the momentum p of a collection of N particles of mass m is:

$$p = -Nmv, \tag{4}$$

The total momentum over a time period of 1 second can be found by considering some parameters for m and n. We consider an object with a surface area of 100 m^2, and we consider that the maximum value for n within a nebula is around 10^{10} particles per m^3 [3]. The masses of the particles can be assumed to be equal to the mass of ionised hydrogen, which is $1.67 \times 10^{-27} kg$. If the collisions are elastic, and the kinetic energy of the particles is conserved, then the change in momentum will be double the momentum of the particles initially. Combining these equations and finding the change in momentum per second, we have:

$$\frac{\Delta p}{\Delta t} = F = \frac{-1.67 \times 10^{-15} \times 2v^2}{1 second} \qquad (5)$$

Since this is proportional to velocity, we can put in some lower velocity values into this equation. Starting with a relatively low velocity value, $v = 3 \times 10^2$, or $v = 300ms^{-1}$. This would give us a value of $F = -3 \times 10^{-10}N$, which is very negligible unless the mass of the spaceship is extremely small. For higher velocities, we will have to consider relativity and relativistic mass. This will only affect the mass of the individual particles with respect to the rest frame of the ship:

$$m = m_0\gamma, \qquad (6)$$

Taking equation (6), we can combine with equation (4) to find:

$$p = -Nm_0\gamma v^2 \qquad (7)$$

This gives us an equation to consider higher velocities, where v is a significant fraction of the speed of light. The relativistic term gamma is defined as:

$$\gamma = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}} \qquad (8)$$

We can consider equation (7) equal to equation (5) multiplied by gamma:

$$\frac{\Delta p}{\Delta t} = F = \gamma\frac{-1.67 \times 10^{-15} \times 2v^2}{1 second} \qquad (9)$$

Results and Discussion

Our results for higher velocity values using equation (7) can be found in the table below:

$v(ms^{-1})$	γ	$\frac{\Delta p}{1s}(N)$
3000	~ 1	-3×10^{-8}
3×10^4	1.000000005	-3×10^{-6}
3×10^5	1.0000005	-3×10^{-4}
3×10^6	1.000050004	-0.03
3×10^7	1.005037815	-3.02
$0.99c$	7.08881205	-2088.5

From our results, it is clear that the resistance of a material as diffuse as a nebula is only not negligible when the speed of an object is close to the speed of light. When it is a large fraction of the speed of light, relativistic effects combine with the effects of the increased speed to create resistances which are high enough to be a huge problem for a spacecraft. For a $100m^2$ spacecraft, you would need a thrust of $2088.5N$ in order to maintain a constant velocity at 99% of the speed of light inside a dense nebula.

Conclusion

Nebulas are the densest part of what can be considered space, and we used the maximum value for the density of a panetary nebula. Despite this, resistance from material is still only a very small force for speeds that are not a significant fraction of the speed of light. However, there are clearly some areas of space and velocities within which you have to "consider air resistance", and nebulas are an object you could expect a future space-faring civilisation to run into while travelling between stars.

References

[1] Shu, F. H. (1985). The physical universe: An introduction to astronomy. Oxford University Press.

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[3] Osterbrock, D. E.; Ferland, G. J. (2006). In Astrophysics of gaseous nebulae and active galactic nuclei (p. 10). essay, University Science Books.