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P4_3 A Ring of Power

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Abstract

Within this paper, we investigate the possibility of a circular platform around the Earth supporting a multitude of wind turbines. The power output of the construct could be described as a function of altitude. It's discovered that the ring produces max power of approximately 130 GW. However, this occurs at an altitude of 0 m, suggesting the platform is not a good idea.

Introduction

A circular platform around the Earth could be filled with wind turbines to generate power for the human population. The ring would be constructed around an assumed perfectly spherical Earth, placed at a set altitude where the power generated would be maximized. The platform would have to be made of an extremely strong material to ensure the ring does not collapse due to its weight.

Method and Equations

To find the maximum power generated by the ring, a graph was plotted of position altitude against power output. As the altitude of the construct increases, the total number of wind turbines that can be placed upon it will also increase. However, the temperature and therefore density of the air would decrease, affecting the power generated by each turbine.

To model this situation, the maximum investigated altitude was 11,000 m. When lower than this altitude, it can be approximated that temperature changes linearly with respect to altitude. Above this altitude the relationship changes where this scenario can not be performed efficiently. Therefore 11,000 m was selected as the maximum altitude for the experiment. The linear relationship below 11,000 m is demonstrated in Eq. (1) [1].

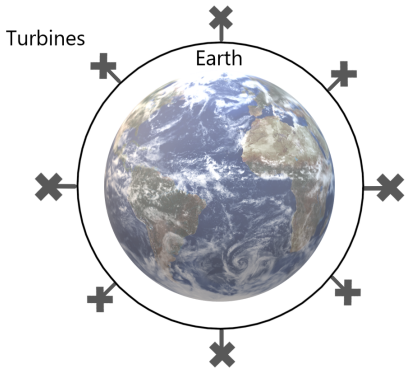

Figure 1: Exaggerated diagram of the circular platform, with turbines installed, spanning the Earth's circumference in only one plane.

$$T = T_0 - Bz \tag{1}$$

The symbols in Eq. (1) are as follows:
T, temperature of air in Kelvin.
T_0, air temperature at the surface of the Earth.
B, lapse rate of value 0.0065 Km^{-1}.
z, position altitude.

Below the 11,000 m boundary, the atmosphere can be stated as ideal. This leads to Eq. (2) and Eq. (3) with the use of Eq [1] and Eq [2] respectively .

$$p = p_0 \left(1 - \frac{Bz}{T_0}\right)^{\frac{g}{RB}} \tag{2}$$

$$\rho = \frac{pT_0\rho_0}{p_0T} \tag{3}$$

where:

ρ, density of air.

p, air pressure.

R, ideal gas constant of value 287 J Kg^{-1} K^{-1} [1].

ρ_0, surface air density.

p_0, surface pressure.

Finally, g is the Earth's gravitational acceleration constant, 9.81 ms^{-2}. This value can be assumed constant below 11,000 m.

Next, Eq. (1), Eq. (2) and Eq. (3) were combined to produce Eq. (4) which shows how the density of air changes with altitude.

$$\rho = \left(1 - \frac{Bz}{T_0}\right)^{\frac{g}{RB}} \left(\frac{T_0}{T_0 - Bz}\right) \tag{4}$$

To calculate the total power produced (P_{total}) we multiplied the power output of one turbine [3], by the number of turbines that could fit upon the ring. The maximum number of turbines could be approximated by dividing the circumference of the ring by the diameter of one wind turbine. The total power produced could therefore be demonstrated below as Eq. (5).

$$P_{total} = \left(\frac{\pi^2 r\eta V^3 \left(R_e + z\right)}{2}\right) \times \rho \tag{5}$$

R_e, radius of the Earth, 6371 km.

r, the radius of a standard wind turbine, 58 m [3].

η, the efficiency of a wind turbine. This experiment assumes the value used is the maximum theoretical value for efficiency, 0.59 [4].

V, average wind velocity in the United Kingdom in 2020 which was 4.63 ms^{-1} [5].

Eq. (4) and Eq. (5) together describe how the total power output relates to position altitude.

Results

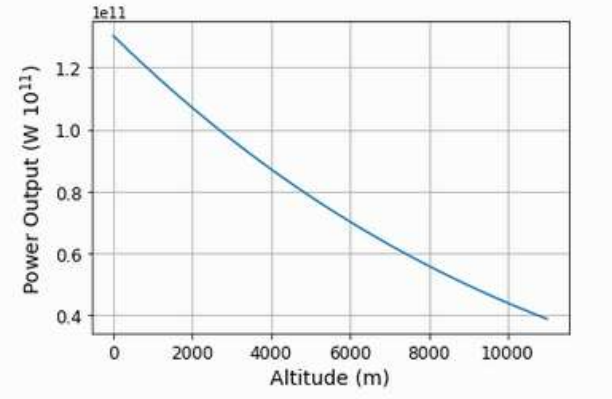

Figure 2: The relationship between total power output and placement altitude.

The maximum possible power generated by the construct was found to be approximately 130 GW. This power was achieved when the ring was positioned at an altitude of 0 m. This suggests that the lowering air density has a more significant effect on the power output compared to the capability of adding more turbines due to the radius of the ring increase.

Conclusion

The experiment showed that the maximum output for the construction was 130 GW. However, this occurs at an altitude of 0 m, indicating the concept does not warrant its construction. If the ring of turbines was built around the Earth at 0 m, the power output of 130 GW would be a significant addition to the current global wind power production of 651 GW [6].

References

[1] `https://bit.ly/3IixjZI`

[2] `https://bit.ly/3Dwf3Is`

[3] `https://bit.ly/31q0QiN`

[4] `https://bit.ly/3EJugqU`

[5] `https://bit.ly/3oPaG62`

[6] `https://bit.ly/31jZlmN`