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P1_3 You chipped the paint

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Abstract

We often hear about space debris threat to satellites. Perspective on the energy of a small object like a paint chip ($\sim$ 1g), which can hold energies approximate to that of a stick of TNT, can highlight the danger. Although most of the energy in an impact is used vaporising the chip.

Introduction

We often hear about space debris, and how even small debris can be incredibly dangerous for spacecraft. In this paper we will attempt to quantify how much energy a relatively small object has when moving at orbital velocities.

We will use a chip of paint as an example, they are quite common as space debris [4] and unless it is lead based, it poses no particular risk to us in our everyday lives.

Theory

To calculate the energy of impact we will assume all the chips are given enough energy to fall back to Earth, without this additional energy the chips would continue on the same trajectory as the spacecraft. Additionally, some unrealistic scenarios are included such as retrograde orbits from Mars, these would require so much energy to be impractical. However these scenarios are included as paint chips are not the only small objects in space, other objects could conceivably be on this trajectory.

For every scenario considered the energy of impact was calculated compared to a satellite in low Earth orbit (LEO) by using equation 1.

$$\triangle E = \frac{m}{2}(v_{chip}^2 \pm v_{leo}^2) \tag{1}$$

The difference in energy is equal to the kinetic energy of the chip from the frame of reference of the satellite.

For chips within the sphere of influence of earth, a variation of the Vis-viva equation, equation 3, can be used to calculate the velocities.

$$v = \sqrt{GM(\frac{2}{r} - \frac{1}{a})} \tag{2}$$

The Vis-viva equation [1]

$$v = \sqrt{2GM_{\bigoplus}(\frac{1}{r_s} - \frac{1}{r_c + r_s})} \tag{3}$$

Where the radius at collision (r_s) is 200km and the semimajor axis (a) is half the sum of the paint chips orbit and the satellite orbit.

For chips starting outside the sphere of influence of earth an additional term needs to be calculated v_∞, this is the additional speed over the escape velocity of earth such that $v^2 = v_\infty^2 + v_{escape}^2$. However, this will be treated as the relative velocity of the paint chip to earth in interplanetary space such that:

$$v_\infty^2 = v_{Interplanetary}^2 \pm v_{Earth}^2 \tag{4}$$

We can assume that v_∞ only contains two terms as to escape orbiting their body of origin (e.g.

Mars) requires energy, but this energy is used to return the object to Mars's orbital velocity around the Sun. From equation 4, $v_{Interplanetary}$ can be calculated using the same variation of the Vis-viva equation as before but now in the sphere of influence of the Sun. Equation 5.

$$v_{Interplanetary} = \sqrt{2GM_{\odot}(\frac{1}{r_{Earth}} - \frac{1}{r_c + r_{Earth}})} \tag{5}$$

V_{Earth} is known. So substituting v_{∞}^2 from equation 4 into equation 6 gives the velocity of a hyperbolic orbit. Equation 6 [1]

$$v = \sqrt{\frac{2GM}{r} + {v_{\infty}}^2} \tag{6}$$

gives us a value for v which we can substitute into our original equation (equation 1) to get the energy of the paint chip.

For Voyager 1 as it has reached the solar systems escape velocity, using $v_{voyager}$ and equation 7, then substituting into equation 8 will give a value of v_{∞} that can be used in equation 6.

$$v_{escape} = \sqrt{\frac{2GM_{\oplus}}{r_{voyager}}} \tag{7}$$

$$v_{\infty}^2 = v_{voyager}^2 - v_{escape}^2 \tag{8}$$

The values of energy are then converted into the equivalent energy of TNT to provide greater context to the values.

Results

Table 1. Retrograde and prograde satellites collisions relative velocity. (Units: kms^{-1})

	LEO	GEO	Luna	Mars	Voyager
Min	0	2.0	3.0	4.0	37
Max	16	18	19	20	53

Table 2. Max and Min relative velocities including retrograde orbits with respect to planets. (Units: kms^{-1})

	Mars	Voyager
Min	4.0	37
Max	65	74

Table 3. Retrograde and prograde satellites collisions relative energy (TNT equivalence). (Units: g of TNT)

	LEO	GEO	Luna	Mars	Voyager
Min	0	0.5	1.0	2.0	140
Max	27	36	38	43	310

Table 4. Max and Min relative energies (TNT equivalence) including retrograde orbits with respect to planets. (Units: g of TNT)

	Mars	Voyager
Min	2.0	140
Max	580	750

Conclusion

The orbital energies of a small object such as a paint chip can be incredibly high, with values exceeding 200g of TNT, or about 1 stick. However, not all of this energy will be transferred to the satellite, it will mostly go into vaporising the paint chip. A thin shell to break these objects up before impacting the main satellite body such as a Whipple shield can remove most of the danger from these objects to large satellites.

Definitions

r refers to orbit radius not planetary radius $m = 0.001\ kg$, $M_{\oplus} = 5.97 \times 10^{24}\ kg$, $M_{\odot} = 2 \times 10^{30}\ kg$, $G = 6.67 \times 10^{-11}\ m^3kg^{-1}s^{-2}$, $v_{Earth} = 2.98 \times 10^4\ ms^{-1}$, $r_{Earth} = 1.49 \times 10^{11}\ m$, $r_{leo} = 6.571 \times 10^6\ m$, $r_{geo} = 4.2164 \times 10^7\ m$, $r_{moon} = 4.00171 \times 10^8\ m$, $r_{mars} = 2.28 \times 10^{11}\ m$, $r_{voyager} = 2.3 \times 10^{13}\ m$[2], $v_{voyager} = 1.0 \times 10^6\ ms^{-1}$ [2], $E_{TNT}\ kg^{-1} = 4184000J$ [3]

References

[1] https://venautics.space/en/complement-2/bases-of-the-mechanical-orbital/3-trajectory-hyperbolic/ [Accessed 16/10/21]

[2] https://voyager.jpl.nasa.gov/mission/status/ [Accessed 16 /10/21]

[3] https://www.unitsconverters.com/en/Kg-Of-Tnt-To-J/Utu-3507-3452 [Accessed 16/10/21]

[4] shorturl.at/bqLO4 [Accessed 29/11/21]