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A1_3 Modelling the Sustain of an Electric Guitar

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Abstract

In this paper we aim to construct a model of the string vibrations and their sustain in solid body electric guitars, considering a driven body oscillation that drains energy from the string and decreases the sustain. We will apply this model to notes close (high E) and further away (low E) from the natural frequency of the body. For the high E, we find a sustain reduced by a factor of two to 1.1 s due to the body, while the low E is essentially unaffected at 7.8 s.

Introduction

The sustain, how long a note sounds, is characteristic of a guitar. In this paper we will develop a simple model to find the sustain of an Ibanez RG570 solid body electric guitar, considering the energy lost from the string due to the vibration of the body. Then, we will examine the impact of the body vibration on the length of the sustain close to and further away from the body's natural frequency.

Theory

First, we consider the solitary movement of a string fixed at both ends. When this string gets plucked, it creates a stranding wave with waveform

$$y = A(t) \sin kx \cos \omega_0 t, \tag{1}$$

where all variables have their general meaning. Since this string is a real system, we assume it experiences damped oscillation such that the energy in the string is given by

$$E_S = E_0 \exp\left(-t/\tau_0\right), \tag{2}$$

where $E_0 = m_S \omega_0^2 A_0^2/2$ is its initial energy (and m_S is the string mass), and $Q = \omega_0 \tau_0$ is the quality factor of the string. 2τ is the sustain, over which time the amplitude decays by e^{-1}.

This string is fixed to a body, which will vibrate in response. The force experienced by the body from the string is given by $F = T\partial y/\partial x$, where T is the tension in the string. If we consider a forced oscillation of the body driven by a force of the form $F = F_0 \cos \omega_0 t$, differentiating Eq. 1 at $x = 0$ gives F_0 as

$$F_0 = TA(t)k. \tag{3}$$

The amplitude of a forced oscillation is $A = F_0/\sqrt{m_G^2 \Delta\omega^2 + b^2\omega_0^2}$, where m_G is the guitar mass, $\Delta\omega$ is the difference of squared natural frequency of the body ω_{0G}^2 and ω_0, and b is its dampening constant. Combined with the energy in an oscillation of $E = m\omega^2 A^2/2$ and the driving force from Eq. 3, the energy lost due to vibration of the guitar, E_G, is

$$E_G = \frac{1}{2} C A^2(t) \text{ and } C = \frac{m_G \omega_0^2 T^2 k^2}{m_G^2 \Delta\omega^2 + b^2 \omega_0^2}. \tag{4}$$

Results

Here we will examine the importance of the energy loss in the string due to the body vibration compared with its own dampening, for the high E and low E strings. We set up an algorithm that calculates the amplitude of the string oscillation as a function of time, after an initial amplitude A_0, energy E_0 of the string, and E_{G0} of the body. For each cycle $T = 2\pi/\omega_0$, the new energy E is found by subtracting the previous cycle's body loss E_G from the previous cycle's final energy, dampened over a time T (Eq. 2). The new amplitude A is inferred from E, which is used to find the new body loss E_G (Eq. 4).

We used two different strings on a scale length of 25.5", the high E and low E strings, which have $f = \omega_0/2\pi = 330\,\text{Hz}, 82\,\text{Hz}$, $m_S = 3 \times 10^{-4}\,\text{kg}, 4 \times 10^{-3}\,\text{kg}$, $T = 72\,\text{N}, 78\,\text{N}$, $k = \omega_0/\sqrt{T/\mu}$ where μ is the mass per unit length, and Q values of approximately 2500 and 2000, which give $\tau_0 = 1.2\,\text{s}, 3.8\,\text{s}$ respectively [1, 2, 3]. For the guitar we use the Ibanez RG570, which has a natural frequency at 325 Hz [4]. Its mass is 3.5 kg and the damping constant can be estimated as $b = 2m_g\omega_{0G}\zeta$ where a standard value of $\zeta = 0.02$ for wood [5, 6]. Fig. 1 shows the amplitude of the string oscillations as a function of time, with the position of e^{-1} at the sustain marked by the dashed line. For the high E string, $2\tau \sim 1.1\,\text{s}$ and for the low E string $2\tau \sim 7.8\,\text{s}$.

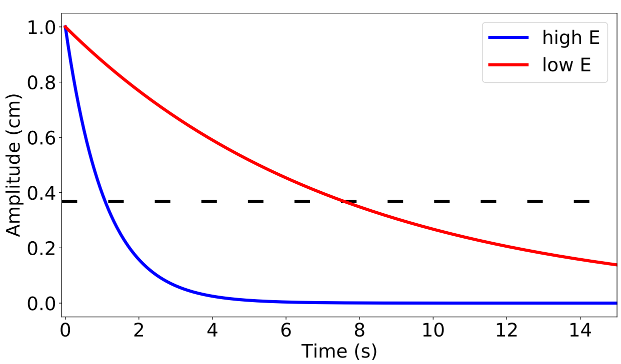

Figure 1: The amplitude of the high E (blue) and low E (red) string as a function of time, including the energy lost due to the guitar body vibrations. The dashed line is at $A(t = 2\tau) = \mathrm{e}^{-1}A_0$.

Conclusion

The importance of the body oscillations on the sustain of a note is highly dependent on the frequency of the note. From Fig. 1, the high E has a final sustain of $\sim 1.1\,\text{s}$, which is half of the original sustain. Therefore, a significant amount of energy was lost due to body oscillations, as the high E is close to the natural frequency of the body. On the other hand, the sustain of the low E is essentially identical to the sustain of the string alone ($2\tau \sim 7.8\,\text{s}$), indicating very little body movement and lost energy. Electric guitars are constructed such that the pickups fixed to the body move as little as possible, to not distort the signal as the string swings. Here we can see that while this works well for frequencies far away from the body's natural resonance, closer there is a significant impact on the note quality. In addition, the energy of the body (Eq. 4) contains a factor of $1/m_G$, meaning less energy will be lost in more massive guitars. This corroborates observations that show that more massive and dense guitar models have longer sustains. While we ignored some more complex aspects of the guitar, such as the neck and shape of the guitar, this model does qualitatively reproduce the basic physical characteristics of solid body guitars.

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