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P4_7 One Ramp To Rule Them All

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Abstract

We explored the possibility of a large stone ramp lined with sand to facilitate the construction of the Great Pyramid, considering a scenario where the ancient Egyptians would have placed blocks on wooden sleighs and used water to lubricate the sand. The optimum angle of the incline of the ramp was found to be 7.2° to the horizontal, stretching about 1163 m horizontally. The construction of such a ramp was found to be possible requiring about 426,000 stone blocks given a ramp width of 5 m.

Introduction

The pyramids of Giza are one of the oldest buildings that are standing to this date and are regarded as one of wonders of the world. The methods used to construct such marvels of engineering with the primitive technology of 2500 BC are not clear, especially considering that the ancient Egyptians did not have access to wheels at that time [1]. Thus, transportation of 2.3 million blocks each weighing on average about 2.5 metric tons to altitudes up to 147 m remains a mystery [2].

In this article we are going to explore the possibility of using a ramp to transport the blocks placed on a wooden sleigh up to the construction level and work out what an optimum ramp would look like.

Theory and Results

To test the ramp theory, we are going to consider the Great Pyramid, which stood at a height of 147m when constructed [2]. By optimising for the force required to move a block up the ramp and the total work required to move the block to the desired height, we can determine the ideal angle of inclination for such a ramp.

By balancing the forces acting on a block on an incline, the minimum force required to move it up the incline, ignoring static friction, is given by:

$$F = mg \times (sin\theta + (cos\theta \times \mu_{\mathrm{d}})). \qquad (1)$$

Where, m is the mass of the object, g is acceleration due to gravity, θ is the angle of inclination of the ramp and μ_{d} is the dynamic coefficient of friction. The average force an adult human can apply by pulling a rope can be estimated to be about 68 kg (588 N) [3], which can be used to calculate the minimum number of people required to move a stone up the ramp by dividing the force required by 588 N.

While the work required to move an object to the top of the ramp is given by:

$$W = F \times d = F \times \frac{h}{sin\theta}. \qquad (2)$$

Where d is the length of the ramp and h is the height of the ramp.

The ancient Egyptians used techniques such as wetting the sand in front of the wooden sleigh, which was first thought to be ceremonial, but experiments conducted showed that this reduces the friction coefficient of the sand by a factor of 3, and the friction coefficient was calculated to be about 0.2 in such a scenario [3]. This reduction in the coefficient of friction is due to increase in the shear modulus of the sand due to the formation of capillary water bridges. [4]

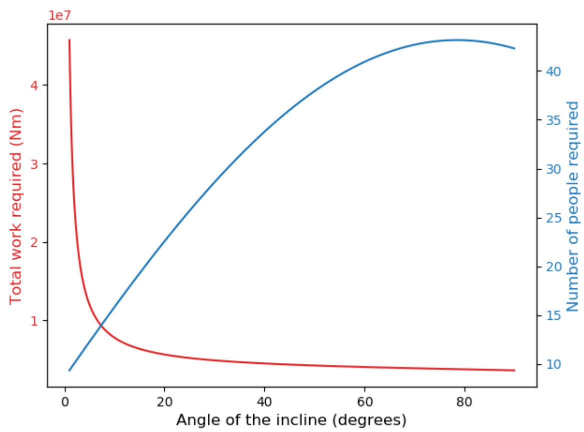

Figure 1: Work required (red) and the minimum number of people required (blue) to transport a 2.5 ton stone up to a height of 147 m as a function of incline of the ramp.

By plotting the work required and the number of people required to move a 2.5 metric ton block to a height of 147 m as a function of inclination of the ramp (Figure 1), we can minimise both functions to find the optimum angle of the incline. This was graphically found to be about 7.2° where the functions intersect.

Discussion

Figure 1 shows that as the angle of incline increases, the total work required to transport the block decreases due to a reduction in the ramp's length while the force required to move it increases. Given an incline of about 7.2°, the ramp would be about 1172 m long and stretch 1163 m long horizontally.

Moving a block up this ramp would require a minimum of about 14 people assuming constant power output, but it in reality it would take more people to do it taking fatigue and the endurance of the human body into account. Also, some additional pushers might be required to overcome static friction and set the block into motion initially.

If we assume the width of the ramp to be about 5 m, then the volume of the ramp would be 426 x 10^3 m^3. The average volume of a limestone block used in the construction of the Great Pyramid was about 1 m^3 [2], which would have taken about 426,000 blocks to construct such a ramp, assuming no spacing between the blocks. Considering that about 2.3 million blocks were used in the construction of the pyramid [2], the construction of such a ramp seems possible by the ancient Egyptians.

Conclusion

Even though this technique is very inefficient compared to modern technologies such as cranes, use of such a ramp is viable and the ramp can be built up slowly along with the Pyramid during construction.

Further work can be done on investigating the feasibility of other theorised stone transport mechanisms such as using a series of spiralling ramps around the Pyramid or using the faces of the Pyramid as ramps.

References

[1] `https://mae.ufl.edu/~uhk/PYRAMID-TECHNOLOGY.pdf`

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