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A4_11 So much potential but, alas, it's a loose wire

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Abstract

In this paper, we determine the potential difference required to disconnect a parallel plate capacitor from a circuit, due to the breaking of its attached copper wires. We find this value to be 3.8×10^5 $\pm$ 0.6×10^5 V. We note that this potential difference would not be possible using air as a dielectric, and a possible solution would be to use water.

Introduction

The plates of a capacitor, when in a live circuit, hold opposite charges of equal magnitude. As a result, the plates exert forces on one another via electrostatic attraction. For typical circuitry these forces are very small [1], however it intensifies as the charge on the capacitor plates increases, which occurs as the voltage across the capacitor is amplified [2]. In this paper, we consider the voltage across the capacitor required for this force to break the wires connecting the capacitor plates to the circuit, which occurs when the force exceeds their tensile strength.

Theory

We model the system as two parallel capacitor plates, surrounded by air, separated by a distance d, each connected to a power supply via uninsulated copper wires. When the circuit is live, the capacitor plates hold opposite charges and hence attract one another via electrostatic attraction. The force acting on each capacitor plate depends on the charge of the opposing plate and the electric field strength generated by this plate. The total electric field strength is given by the sum of the electric field strengths generated by each plate [3]. As the magnitudes of charge on each plate are equal, the electric field strength generated by each is half the total electric field strength between the capacitor plates. As a result, the magnitude of the force acting on each capacitor plate, F, is found using Equation (1). In this equation, Q is the magnitude of the charge on each plate and E is the total electric field strength between the capacitor plates.

$$F = \frac{1}{2}EQ \tag{1}$$

As we wish to relate the force on the plates to the voltage across the capacitor, we replace E using Equation (2). In this equation V is the voltage across the capacitor and d is the separation of the capacitor plates.

$$E = \frac{V}{d} \tag{2}$$

By considering the capacitance of the capacitor, we may replace the charge in Equation (1) with another term including the voltage, this is shown in Equation (3). In this equation, C is the capacitance of the capacitor, which is calculated using Equation (4).

$$Q = VC \tag{3}$$

$$C = \frac{\kappa \epsilon_0 A_p}{d} \tag{4}$$

In Equation (4), ϵ_0 is the permittivity of free space, A_p is the area of the capacitor plates and κ is the relative permittivity of the dielectric. By replacing E and Q in Equation (1) using Equations (2) and (3) and replacing C using Equation (4), the magnitude of the force that the capacitor plates exert on each other is given by Equation (5).

$$F = \frac{\kappa \epsilon_0 A_p V^2}{2d^2} \tag{5}$$

The stretching stress required to break a material is given by its "tensile strength" [4], this is given by Equation (6). In this equation T_s is the tensile strength of the material, F_S is the stretching force applied and A is the cross-sectional area of the material.

$$T_s = \frac{F_S}{A} \tag{6}$$

By rearranging Equation (6) for the stretching force and equating to Equation (5), we can find the voltage required to break the wires connected to the capacitor plates, assuming that the force acting on the them also acts on the copper wires that connect them to the circuit. This is shown in Equation (7).

$$V = d\sqrt{\frac{2T_s A}{\kappa \epsilon_0 A_p}} \tag{7}$$

Results

The tensile strength of copper is 300 ± 90 MPa [5] and the relative permittivity of air is $\kappa = 1.0006$ [6]. We use the values of a standard parallel plate capacitor as stated in [4]. These are: a plate separation of $d = 1$ mm, copper wire cross-sectional area of $A = 21.15$ mm^2 and a capacitor plate area of $A_p = 0.01$ m^2 [4]. These are assumed dimensions and so there are no uncertainties in these values. Using these values, the voltage required to break the copper wires connecting the capacitors to the power supply is $3.8 \times 10^5 \pm 0.6 \times 10^5$ V.

By substituting this voltage into Equation (2), this results in an electric field between the plates of $3.8 \times 10^8 \pm 0.6 \times 10^8$ V m^{-1}. This value far exceeds the electrical breakdown of air, 3×10^6 V m^{-1}, meaning that an air dielectric would be unable to support the field required. A potential alternative dielectric for this system is water. As $\kappa \approx 80$ for water at room temperature [7] the electric field between the plates at which the wires are broken is reduced to $4.2 \times 10^7 \pm 0.1 \times 10^7$ V m^{-1}. The dielectric strength of water is 7×10^7 V m^{-1} and so this could support the required electric field.

Conclusion

In this paper, the voltage across a capacitor (of defined dimensions) required to break the uninsulated copper wires which connect each capacitor plate to the power supply has been determined as $3.8 \times 10^5 \pm 0.6 \times 10^5$ V. This would be unobtainable when using an air dielectric, as the breakdown electric field of air is far below the electric field strength that this voltage induces across the capacitor $3.8 \times 10^8 \pm 0.6 \times 10^8$ V m^{-1}. This should however be possible by using a water dielectric, as this reduces the required electric field strength to $4.2 \times 10^7 \pm 0.1 \times 10^7$ V m^{-1} and is capable of supporting up to 7×10^7 V m^{-1} before dielectric breakdown. Other issues with the system, including the feasibility and safety elements of uninsulated copper wire supporting such a high voltage, though these details are beyond the scope of the paper.

References

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