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A2_9 Raindrops keep falling on my head

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Abstract

In this paper we attempted to find the density need for an object of raindrop size to kill a human if it were to land on their head. We concluded that the density of titanium, 4500 kg m^{-3}, is effectively to the lower limit that would be able to fracture the skull.

Introduction

Most of us have probably heard that if a penny were dropped from a skyscraper that it could kill you. The idea behind this is that as the penny builds up speed the amount of force it imparts breaks through your skull enough to damage your brain irreparably. While it seems like a sensible idea, dropping a penny from a skyscraper would not kill you [1]. The penny does not build up speed until it collides with your head, it reaches it's terminal velocity long before that. In this paper we will use the size of raindrops and find the density required to for something the size of a raindrop to damage the skull. For this the shape of a raindrop is being approximated as a sphere.

We will be looking at what density is required to fracture the skull and thus enable the brain to be damaged directly, hypothetically leading to death.

Theory and Equations

The dimensions of a raindrop with radius, $r = 2$ mm are mass, $m = 3.4 \times 10^{-5}$ kg and a cross-sectional area, $A = 1.3 \times 10^{-5}$ m^2 [2] and volume, $V = 3.35 \times 10^{-8}$ m^3. Due to approximating the raindrop as a sphere when calculating the terminal velocity we can say that its drag coefficient, $D = 0.5$. Under normal circumstances the velocity of a raindrop can be said to be 9 ms^{-1} [2].

The terminal velocity, v_t, of an object can be calculated using the following equation (1), where all variables are as above and g is the gravitational field strength of Earth and ρ_a is the density of air, which is $\rho_a = 1.32$ kg m^{-3} at 20°C [3].

$$v_t = \sqrt{\frac{2mg}{D\rho_a A}} \tag{1}$$

The fracture energy of bone is $G_c = 1.5$ kJ m^{-2} [4], therefore we are able to use a slightly modified formula of kinetic energy, E_k (2)

$$E_k = \frac{1}{2}mv^2 \tag{2}$$

to solve this problem. As the fracture energy of bone has an m^2 term we are simply able to multiply the the fracture energy of bone by the cross-sectional area of the raindrop, A, as can be seen in equation (3) below:

$$G_c A = \frac{1}{2}mv_t^2, \tag{3}$$

which gives us a new value of $G_c = 0.0195$ J.

Results

With these equations we are now able to start putting data into the equations. Firstly let us try the most dense element and see if that is able to fracture the bone. Osmium is the most dense element and at room temperature (20°C) its density is 22,590 kg m^{-3} [5]. Therefore using the density formula, $\rho = m/V$, we find that a raindrop sized piece of osmium has a mass of 7.57×10^{-4} kg and so its terminal velocity is 41.6 ms^{-1} which leads to a kinetic energy of 0.66 J, which is $\sim$ 30 times larger than the energy needed to fracture 1.3×10^{-5} m^2 of bone that was calculated above.

Using this same process this we are able to find the same data for many other elements as can be seen below in table 2:

Table 1: Table of elements and their results [6]

Element	ρ (kg m^{-3})	m (kg)	v_t (m s^{-1})
Silver	10490	3.5×10^{-4}	28.3
Zinc	7135	2.4×10^{-4}	23.4
Titanium	4500	1.6×10^{-4}	19.1
Barium	3594	1.2×10^{-4}	16.6

Table 2: Comparing found fracture energy to G_c

Element	Fracture Energy (J)	Ratio to G_c
Silver	0.14	~ 7
Zinc	0.065	~ 3
Titanium	0.027	~ 1
Barium	0.017	~ 0.9

Discussion and Conclusion

From the data above we can conclude that elements more dense than titanium be able to fracture the skull and that it is or at least close to the lower limit. However while we know that titanium would be able to fracture the skull we do not know how much other damage titanium would be able to inflict.

That being said we can reasonably assume that the more dense the material the more damage it would inflict on any given skull. It may be safe to assume that silver or certainly osmium would likley penetrate the bone completely and thus damage the brain which would in all likelihood lead to death. Barium it would seem is an element that we would not expect to fracture the skull, though it would likely break the skin and cause a lot of pain.

We choose metals we believed to be well known, except for osmium which was choosen due to it being the densest element.

Of course this model is for within Earth's atmosphere and on other planets or moons the results would vary quite severely.

Acknowledgements

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References

[1] https://www.livescience.com/18832-penny-dropped-skyscraper.html

[2] R. A. Serway and J. W. Jewett Jr., *Physics for Scientist and Engineers with Modern Physics*, (Brooks/Cole, 2014), 9th Edittion p. 165, table 6.1

[3] https://www.engineeringtoolbox.com/air-density-specific-weight-d_600.html

[4] https://www.doitpoms.ac.uk/tlplib/bones/bone_mechanical.php?fbclid=IwAR2qbGb2uwJ7Z-_Sl3TSehs246MFYkVA1yf5VXa8102PjthcM4J2hGlB9

[5] https://www.technology.matthey.com/article/33/1/14-16/

[6] https://www.engineeringtoolbox.com/metal-alloys-densities-d_50.html