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P2_9 Torque about a Cliffhanger

K. Golsby, H. Biddle, K. Hinchcliffe, J. Mooney

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

In this paper we attempt to calculate the amount of overhang of the coach in the final scene of 'The Italian Job', after it skids to a stop over the edge of a cliff, and found the value to be 3.7 m. We also found a relation between the acceleration of the angle of tilt of the coach, and the distance moved by a man initially at one end of the coach to generate a non-zero torque.

Introduction

The film 'The Italian Job' ends on a very literal cliffhanger. After a heist which gains the group $4 million in gold [1], they find themselves skidding round rather windy moutain roads, until they lose control and their coach ends up resting on a cliff edge with the gold hanging over the edge. The film ends on this note, leaving the audience guessing. By estimating the mass and positions of the contents of the coach, the pivot point at which the coach will be balanced on the very edge of the cliff can be found. Using this, the relationship between the angle of tilt of the coach over time with the distance the main character moves as he tries to save the gold can be found.

Method

In order to find the position of the pivot point along the stable coach such that the moments created by the group of people standing far from the cliff edge and that from the gold overhanging the edge are balanced, the total clockwise and total anticlockwise moments must be equal. We have assumed that the pivot point is fixed, and that the coach does not slide down the cliff, and that the gold has slid to the very end of the coach. The moments on either side of the pivot point are given by equation 1, in which acceleration due to gravity has been cancelled given $F = mg$.

$$m_g x_1 = m_{com} x_3 + m_p(x_2 + x_3), \tag{1}$$

where the mass of the gold, m_g, is 3200 kg [1], the centre of mass of the coach m_{com} is 3200 kg [2], and the mass of the group of people m_p is 840 kg, assuming that each of the twelve people (as seen in the film) are 70 kg [3] in mass. The total length L of the coach, estimated to be 11 m [2] is given by:

$$L = x_1 + x_2 + x_3, \tag{2}$$

where x_1 is the distance from the gold to the pivot point, x_2 is the distance from the people to the centre of mass of the coach, and x_3 is the distance between the centre of mass and the pivot point. As the centre of mass of the coach lies at $\frac{1}{2}L$, it can be seen that,

$$x_1 + x_3 = \frac{1}{2}L = x_2. \tag{3}$$

Combining and rearranging equations 1 and 3, the distance x_1 can be found,

$$x_1 = \frac{L}{2}\left(\frac{m_{com} + 2m_p}{m_{com} + m_p + m_g}\right), \tag{4}$$

showing the position of the pivot point to be ∼3.7 m along the length of the coach (x_1) i.e. there is ∼3.7 m overhang.

We will now find a relationship between the second derivative of the angle of tilt of the coach and the distance travelled by Michael Caine as he inches towards the gold as in the film. Any movement results in equilibrium being lost and the coach will begin to tip, due to the resultant torque generated, as the equation for torque due to a given force shows:

$$\tau = F_{\perp}L = I\alpha = I\frac{d^2\theta}{dt^2}, \tag{5}$$

where L is the perpendicular distance between the pivot point and the force F, I is the moment of intertia of the coach, and α (the second derivative of θ) is the angular acceleration of the coach. Considering the contributions to the net torque $\tau = 0$ at equilibrium, we can show:

$$\tau = W_G cos\theta x_1 - W_{CoM} cos\theta x_3 \\ - W_{man} cos\theta(x_2 + x_3) - W_{p-1} cos\theta(x_2 + x_3) = 0, \tag{6}$$

where W_{G} is the weight of the gold, W_{CoM} is the weight of the coach, W_{man} is the weight of one man, and W_{p-1} is the weight of the rest of the men. These weights, each multiplied by $cos(\theta)$, are synonymous to the value $F_{\perp}$ in equation 5. $\theta = 0$ in this case as the coach is level. As Michael Caine (whose weight is represented by W_{man}) moves, there is a change in the contribution of these forces to the torque, like so:

$$\tau = W_g cos\theta x_1 - W_{CoM} cos\theta x_3 \\ - W_{man} cos\theta(x_2 + x_3 + \delta x) - W_{p-1} cos\theta(x_2 + x_3), \tag{7}$$

where θ is now changing with δx, the distance moved by Michael. Cancelling down the like-terms in these two equations gives a change in torque of

$$\delta\tau = W_{man} cos\theta\delta x, \tag{8}$$

which allows us to find function for the rate of tipping, i.e. acceleration of θ, by combining the torque and the angular rotation of the coach about the pivot point (equations 5 and 8):

$$\frac{d^2\theta}{dt^2} = \frac{W_{man}}{I} cos\theta\delta x \propto cos\theta\delta x. \tag{9}$$

Conclusion

In this paper we have calculated the amount of overhang the coach experiences (i.e. the distance to the ledge described here as a pivot) in the final scene of 'The Italian Job' to be 3.7 m. We also found a result for the relation between the acceleration of the angle of tilt of the coach and the distance moved by a man initially at one end of the coach generating a non-zero torque. This acceleration (the second derivative of θ) depends also on the value of θ at any given point meaning any tilt of the coach will result in a runaway effect whereby the coach continues to tilt at an increasing rate, so saving the coach at this point would be virtually impossible. Assumptions have been made to simplify this scenario such as the lack of tilt initially as the coach finds equilibrium, and the pivot point being fixed, which assumes high enough frictional force between the coach and the ledge that it does not slide- this aspect of the scenario could be investgated further.

References

[1] https://tinyurl.com/qn72bvz

[2] http://www.thcoachwork.co.uk/legion.htm

[3] https://hypertextbook.com/facts/2003/AlexSchlessingerman.shtml