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P3_6 Space Cannon

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Abstract

In this paper, we investigate the possibility of firing spacecraft into orbit using a cannon (likely either a coilgun or railgun). We discover that a cannon capable of doing such things would have to fire the craft at around 8.2 km s^{-1}, and be at least 1142 km long, to avoid placing the craft's inhabitants under fatal accelerative forces.

Introduction

Currently, spacecraft require immense quantities of fuel to get into orbit. The Saturn V rocket, in particular, burned through 20 tons of fuel per second, at its peak [1]. In the interest of resource preservation, as well as other concerns, scientists have been looking into ways to circumvent this issue. The two main ideas are that of the space lift, or elevator, and the space cannon.

Theory

The equation for escape velocity is:

$$v_{esc} = \sqrt{\frac{2GM}{R}}. \tag{1}$$

Using

$$v^2 = u^2 + 2as, \tag{2}$$

it can be seen that:

$$as = \frac{GM}{R}. \tag{3}$$

where a is the acceleration required to accelerate an object from 0 to escape velocity over distance s, the length of the cannon. Interestingly, this implies that, if one starts at the centre of a spherical body and accelerates constantly toward its surface at the acceleration due to gravity at that surface, when the surface is reached, one's speed will precisely equal the body's escape velocity. This is problematic because, even if the cannon accelerates the craft at $3g$ (as astronauts are trained for), it would still have to be one third the radius of the Earth in length. We can shorten this distance considerably, we hope, by launching the craft into low-Earth-orbit (with orbital radius roughly equal to $7\text{x}10^6$ m), instead of having it reach escape velocity. The work done against gravity to move an object from a point, R_1, above the Earth to some other point, R_2, is equal to:

$$W = GMm\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \tag{4}$$

where M is the mass of the Earth and m is the mass of the object. By equating work done to kinetic energy and simplifying, we get:

$$v^2 = 2GM\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \tag{5}$$

for any object with only a vertical component of velocity. If we assume that the Earth is a flat plane, we can create a helpful approximation of how a cannon would fire an object into orbit: if the cannon fires the object at a velocity, v, then it will have vertical and horizontal components of velocity, $v \sin \theta$ and $v \cos \theta$, respectively, where θ is the inclination of the cannon. The idea is that, once the craft reaches its maximum height, determined by $v \sin \theta$, it will have a horizontal velocity, $v \cos \theta$, equal to the orbital velocity at that height. In reality, this approximation is inaccurate, because the Earth is a spheroid, and so a horizontal component of velocity tangential to its surface will, as the object ventures further from its launch point, cause a vertical displacement when viewed radially. Nevertheless, we can assume that the craft has propulsion systems on board to correct its trajectory, once it is in the vicinity of the desired orbit. Additionally, this approximation becomes more true, the closer to 0° or 90° the firing angle is, due to less interference from the vertical or horizontal components of velocity, respectively. With that said, from Equation 5:

$$(v \sin \theta)^2 = 2GM \left(\frac{1}{R_1} - \frac{1}{R_2} \right), \tag{6}$$

and we set the horizontal component to equal the orbital velocity at R_2:

$$v \cos \theta = \sqrt{\frac{GM}{R_2}}. \tag{7}$$

Dividing Equation 6 by the square of Equation 7 gives:

$$\tan^2 \theta = 2R_2 \left(\frac{1}{R_1} - \frac{1}{R_2} \right), \tag{8}$$

$$R_2 = \frac{R_1}{2} (\tan^2 \theta + 2). \tag{9}$$

Additionally, Equation 7 can be rearranged to:

$$v = \sqrt{\frac{GM}{R_2 \cos^2 \theta}}. \tag{10}$$

These equations have three unknown variables and so cannot be solved algebraically, or by using a 2-dimensional graph. Because of this, we created Table 1.

Firing Angle / Degrees	Orbital Distance / 10^6 m	Initial Velocity / km s^{-1}
5	6.395	7.925
10	6.470	7.970
15	6.600	8.045
20	6.792	8.151
25	7.064	8.288
30	7.433	8.456
35	7.933	8.653
40	8.614	8.880
45	9.557	9.133
50	10.90	9.409
55	12.87	9.703
60	15.93	10.01
65	21.02	10.30
70	30.42	10.58
75	50.74	10.83
80	108.8	11.02
85	422.5	11.14
90	∞	11.19

Table 1: Array of possible values for R_2 and v, at various values of θ.

Conclusions

It can be seen from Table 1 that a low-Earth orbit, similar to the ISS, could be achieved with a firing angle of between 20° and 25° giving an orbital radius close to to $7 \text{x} 10^6$ m. However, this still requires that the craft be accelerated to roughly 8.2 km s^{-1}. If we assume again that it accelerates the craft at $3g$, the highest human beings can reasonably withstand for any length of time, the cannon would still have to be 1142 km long, which is ridiculously large. Thus, though the feasibility of a process to fire spacecraft into orbit from the Earth's surface remains an open question, the implementation of such a process would require a design radically different to a straightforward cannon.

References

[1] https://www.space.com/38720-nasa-saturn-v-rocket-surprising-facts.html [Accessed 19 November 2019]