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A4_10 Nobody wants a flat beer

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Abstract

In this paper, we determine the probability for a carbon dioxide molecule to quantum tunnel through a glass beer bottle. The transmission probability was found to be in the range $10^{-e^{1.14\times10^{14}}}$-$10^{-e^{1.28\times10^{14}}}$, depending on the value used for the potential of the glass beer bottle. Even when considering all the gas molecules in the bottle (2.05×10^{22}), the probability of any molecule tunnelling through the bottle is infinitesimal, and can be ignored for all intents and purposes.

Introduction

Most beers are carbonated, however, if they are opened they will slowly de-gas and become flat. From quantum mechanics, we know that particles can tunnel through finite potential barriers, which seemingly contradicts our macro-scale life. In this paper, we will determine the probability for a carbon-dioxide molecule to quantum tunnel through a glass beer bottle.

Theory

To begin the theory, we will talk about the physics of quantum tunnelling (QT). Equation (1) shows the formula for QT for a thick potential barrier. We have used a *thick* barrier as the ratio of the size of a molecule to the thickness of the bottle wall is small. In Equation (1): T is the transmission probability, E is the energy of the particle, V_0 is the potential of a barrier, α is a decay constant and b is the thickness of the barrier.

$$T = \frac{16E\left(V_0 - E\right)e^{-2\alpha b}}{V_0^2} \text{ [1]} \qquad (1)$$

By stating that the bottle is made from glass, we may assume that the barrier is made from SiO_2, as this is a major component of glass [2]. SiO_2 has an molar mass of 60.08 g mol^{-1} [3], which allows us to use the assumption that the potential of the bottle varies between 20-25 MeV [4].

Next, we can determine the value of b. The thickness of a beer bottle varies from 0.20-0.36 cm [5] and so we use the median of this (0.28 cm) as our value for b. For the model, we shall assume that the bottle has a uniform thickness and shape. This is a limitation of the model, but as the thickness and geometry of *standard* bottles is unknown to us, we must make this assumption.

Source [5] used a 0.5 l Feldschlösschen branded beer, which is a *blonde* style of beer. We do not know the volume of carbonation that this particular beer has, but blonde beers can vary in volume of carbonation values from 2.0-4.0 Vol_{CO_2} [6]. We will take the middle value of this, 3.0 Vol_{CO_2}, which would be equivalent to a German weizen (wheat-beer). We therefore know that the volume of CO_2 in this beer is 1.5 l. We will assume that the viscosity of the beer does not affect the speed of the gas particles inside the bottle. One could picture this as an empty bottle, so that the standard gas laws apply. Without conducting experiments to determine variables, we are unable to determine the speed of the dissolved gas particles accurately. However, if we assume that the gas moves only perpendicular to the direction of the beer bottle walls (ignoring diagonal directions) we can work out the speed of the gas particles (v) at room temperature. Making this assumption simplifies the model greatly, but is not very realistic. The result of this assumption would cause the mean time between collisions of the wall and the gas particles to decrease. Future work could find the speed of the particles without this assumption. The speed of the molecules is shown in Equation (2) where: k is Boltzmann's constant, T_e is temperature and m_{CO_2} is the mass of a CO_2 particle. The two in the equation appears as CO_2 has four degrees of freedom [7].

$$v = \sqrt{\frac{2kT_e}{m_{CO_2}}} \tag{2}$$

Setting our temperature to 300 K and knowing the molar mass of CO_2 is 44.01 g mol^{-1} [8] we get a velocity of 337 m s^{-1}. The energy of the particle, using Equation (3), is 51.7 meV.

$$E = 2kT_e \tag{3}$$

If we use a diameter of the bottle of 7 cm [9], the time between particle-wall interactions is 0.21 ms.

The value of α can be found using Equation (4), where $\hbar$ is the reduced Planck constant. When we use a value of V_0 of 20 MeV, the lower boundary for the potential, α is 2.05×10^{16} m^{-1}.

$$\alpha = \sqrt{\frac{2m_{CO_2}}{\hbar^2}(V_0 - E)} \text{ [1]} \tag{4}$$

Results & Discussion

Substituting our values into Equation (1), we get a transmission coefficient of approximately $10^{-e^{1.14\times10^{14}}}$. A 0.5 l beverage at 3.0 Vol$_{CO_2}$ would have 2.05×10^{22} CO_2 molecules (found by using molar theory). If we divide the transmission coefficient by the time between collisions with the bottle, then multiply by the number of molecules of CO_2 we find the probability that any of the CO_2 molecules quantum tunnel through the glass every second. Even when including this, the probability of transmission of the CO_2 molecules is barely affected, due to the high exponent values. The time calculated earlier would be smaller than the *true* value, so if the *true* value was used it would cause less of a decrease to the probability. We used the lower boundary for the potential well, if we used 25 MeV our transmission coefficient would be $10^{-e^{1.28\times10^{14}}}$. This means that most people could consider CO_2 to not be QT through the bottle.

Conclusion

We determined that the transmission of CO_2 molecules through a glass bottle to be in the range $10^{-e^{1.14\times10^{14}}}$-$10^{-e^{1.28\times10^{14}}}$. This is an infinitesimal probability, therefore one can rest assured if a bottle of beer is left unopened, the quantum mechanical affects of tunnelling can be assumed to be insignificant to the de-gassing of the beverage.

References

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