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P2_5 Jack and the Decompression Sickness

K. Hinchcliffe, K. Golsby, H. Biddle, J. Mooney

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

In this paper, we calculate the volume of supplemental oxygen that Jack would need to climb and descend the entire height of the beanstalk in the classic fairy-tale, Jack and the Beanstalk. We also find the pressure of the air at the beanstalk peak and compare this to the pressure at sea-level. These values were found to be 1880 litres and 0.365 Pa respectively.

Introduction

In the classic fairy-tale Jack and the Beanstalk, Jack climbs the Giant's beanstalk to a land 'high in the sky'. The beanstalk reached to heaven in the fable, and therefore assuming an altitude of the Karman Line, the volume of oxygen needed to climb both up and down the beanstalk can be found. The pressure at the peak of the beanstalk can also be found and health effects at this pressure can be discussed.

Theory and Results

In Jack and the Beanstalk, the Beanstalk is said to have reached to heaven, so we have assumed this is the boundary between the atmosphere and space: the Karman Line. We find this altitude to be 100 km from the Earth [1]. At the start of Jack's climb, we can assume he uses oxygen in the air for respiration. However, at a certain altitude known as the 'Death Zone', the oxygen in the air is no longer available. At this point, supplemental oxygen would need to be used to continue the climb. The 'Death Zone' occurs at an altitude of 8000 m [2], therefore meaning Jack would be able to climb 8% of the beanstalk before needing supplemental oxygen to continue the 92000 m journey up.

We then find the time it would take Jack to climb the height of the beanstalk. By using the speed climbing record of 5.48 s to climb 15 m [3], we find that it would take 10.1 hours to climb the entire height of the beanstalk, and 9.33 hours to climb the 92000 m with supplemented oxygen.

As the average person breathes roughly 8 litres of air per minute [4], and oxygen is roughly 21% of this air, we find Jack would require 1.68 litres of oxygen per minute. We therefore find that to complete the 92000 m journey up the beanstalk, Jack would need 940 litres of oxygen. This therefore means a total of 1880 litres of oxygen would be needed for both the climb up and climb down the beanstalk, roughly equivalent to 104 18-Litre standard capacity oxygen cylinders [5].

We can then find the pressure at the peak of the beanstalk, the scenario of which is seen in Figure 1.

Assuming a horizontal disc-shaped element of air, an equation for the pressure can be found in Equation 1,

$$PA - (P + dP)A - mg = 0, \qquad (1)$$

Where P is the pressure at the peak of the

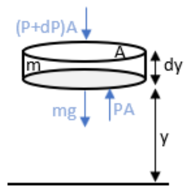

Figure 1: The scenario to show the variables involved in the calculation of the pressure at the peak of the beanstalk.

beanstalk, A is the cross-sectional area of the disc, dP is the difference in pressure between the top and bottom of the disc, m is the mass of the air and g is acceleration due to gravity. The equation is set equal to zero as, when applying Newton's second law to the disc, acceleration is zero and so the sum of the forces equal zero.

Equation 1 can then be simplified, and m can be substituted for $\rho A dy$ (as $m = \rho V$) where ρ is the density of the air at the peak of the beanstalk and dy is the thickness of the disk. Assuming the density is proportional to the pressure we can use $\rho/P = \rho_0/P_0$ where ρ_0 and P_0 are the density of air and pressure at the base of beanstalk respectively. We can rearrange and substitute ρ into Equation 1, giving us Equation 2,

$$\frac{1}{P}dP = -\frac{\rho_0}{P_0}g dy. \tag{2}$$

Integrating both sides of Equation 2 and rearranging for P, we find Equation 3:

$$P = P_0 \exp -\frac{\rho_0}{P_0} g y, \tag{3}$$

where y is the height of the beanstalk. Substituting the pressure and density of air at the base of the beanstalk to be 101×10^5 Pa (1 atm) and 1.29 kg m^{-3} respectively, g to be 9.81 N kg^{-1} and y to be 100 km, we find P to be 0.365 Pa. This is a pressure change of 2.76 Pa s^{-1} that Jack experiences while climbing the beanstalk. The lowest pressure that a human can withstand before explosive decompression is the Armstrong limit of 6300 Pa armstrong, therefore Jack would require a pressurized suit to complete the climb.

Discussion

In this paper, assumptions have been made regarding the conditions experienced by Jack while climbing. The Karman Line falls within the thermosphere where temperatures reach 2000 °C However at this altitude, the air is so thin that there are very few air particles capable of transferring this temperature. For this reason, we assume that Jack would be able to withstand the temperature at the peak. We have also assumed Jack would acclimatize to the altitude quickly, much like a professional climber, so a normal breathing rate could be used when considering the volume of oxygen needed to complete the climb. Further investigation into the effects on the body at this height could be performed. Finally, we have assumed Jack to be a speed climber, therefore giving a minimum time to climb the beanstalk and hence, a minimum amount of supplemental oxygen needed.

Conclusion

In order for Jack to climb the Beanstalk as told in the fairy-tale Jack and the Beanstalk, he would require a pressurized suit (due to the pressure change of 2.76 Pa s^{-1} while climbing) and 1880 litres of oxygen. Jack would complete the climb in 10.1 hours (20 hours round-trip).

References

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