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A4_4 I spy with my telescope eye...

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Abstract

In this paper, we determine if a telescope could be used to identify trade ships entering port two hours earlier than a naked eye, as was claimed by Galileo. We find the difference in resolvable distances between a telescope and a naked eye to be 35 km. By then considering the speed of a 17th century trade ship, we find the time difference as 2.36 hours, similar to Galileo's claim. We have ignored the effect of aberrations caused by imperfect lenses, which may significantly impact this result.

Introduction

Galileo first used his telescope as a method of identifying ships entering port [1]. He claimed that it could be used to identify trade ships entering port two hours before other observers [2]. As it was unknown whether trade ships would arrive safely, due to the danger of being raided or lost at sea, merchants would only purchase a ship's cargo when they could see it entering port. Because of this, being able to identify a trade ship before other observers would give the user of the telescope a large head start in purchasing its cargo. To verify Galileo's claim, we use Rayleigh's Criterion to calculate the maximum distance at which an observer could identify features on the flag of a ship, with and without Galileo's telescope. We then calculate how long a ship would take to cover this distance, to determine if this matches Galileo's claim.

Theory

For light diffracting through a lens with a circular aperture, if the lens diameter D is much larger than the wavelength of light from the source λ, then we may apply the small angle approximation. As shown in equation (1).

$$\sin(\theta) \approx \tan(\theta) \approx \theta \tag{1}$$

The first diffraction minimum (θ) is then given by equation (2).

$$\theta \approx 1.22\frac{\lambda}{D} \tag{2}$$

If the angular separation of two light sources is less than θ, the two sources will be unresolvable. If the distance between the sources, d, is far smaller than the distance from the sources to the lens L, then θ can similarly be represented as the ratio of these values, as shown in equation (3).

$$\theta_c \approx \frac{d}{L} \tag{3}$$

By equating equations (2) and (3) and rearranging for L, this gives the maximum distance at which the trade ships can be identified. This is shown in equation (4).

$$L \approx \frac{dD}{1.22\lambda} \tag{4}$$

A diagram of the system is shown in figure 1.

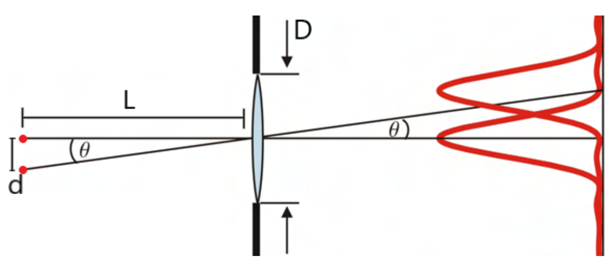

Figure 1: Rayleigh Criterion for the resolution of two points [3]

It is unknown to us how trade ships were identified by observers during this period. We assume that observers identified ships through recognising details on the ship's flag and hence must resolve features $d = 5$ cm apart. We assume a wavelength of $\lambda \approx 500$ nm as this is a typical value in the visible range.

For the ordinary observer we model the pupil as a convex lens, and assume a diameter $D = 5$ mm [4]. Galileo's telescope consisted of a long pipe, with a 37 mm diameter plano-convex objective lens of focal length $f = 980$ mm at one end, and a 22 mm diameter plano-concave eyepiece of focal length $f = -50$ mm at the other [5]. A diagram of a telescope of this description is shown in Figure (2).

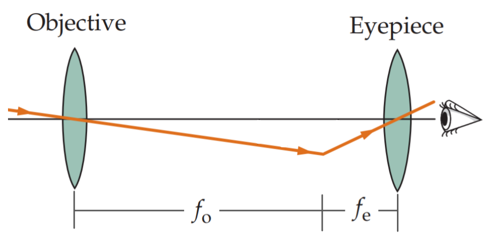

Figure 2: Diagram of a telescope [4]

As the source is far from the telescope, the lens forms an image at a distance of f_o down the pipe. For the user to observe a clear image the eyepiece is then placed a distance of f_e from this image. This process magnifies the image, with the magnification factor M found using equation (5).

$$M = -\frac{f_o}{f_e} \tag{5}$$

For Galileo's telescope, $M = 19.6$. This magnification acts to increase the observed size of the flag, making it easier to resolve details on it. Hence, we multiply d by this magnification factor, giving $d = 98$ cm. Assuming that all light entering the lens also enters the eye, then the diameter applied is that of the lens $D = 37$ mm.

Results and Discussion

Substituting in the forms of d, D and λ into equation (4) gives $L \approx 400$ m for the ordinary observer, and $L \approx 60$ km for the telescope. However, this maximum resolved distance is limited by the distance to the horizon, which is given by equation (6) from source [6].

$$D = \sqrt{2R_E h} \tag{6}$$

Where h is the elevation of the observer and $R_E = 6371$ km is the radius of Earth. Galileo displayed his telescope at the top of the Campanile di San Marco, giving $h = 98.6$ m [7]. This gives the distance to the horizon as 35.5 km, limiting the range of the telescope as it is unable to observe beyond the horizon. The gap between these two maximum distances is then ≈ 35 km, requiring a maximum speed of 17.5 km hr^{-1} to prove Galileo's claim. The Galleon, a typical 17th century trader ship, was observed as travelling at around 8 knot or 14.82 km hr^{-1} [8], meaning that the ship covers this distance in 2.36 hours, suggesting Galileo's claim has merit.

Error in Calculation

We have assumed that the lenses used in Galileo's telescope are perfect, however, his lenses suffered from spherical aberrations [9]. This causes light rays striking the edge of the lens to be deflected at a larger angle than those striking the centre, meaning not all light rays focus on the same point. Details in this image would be much harder to resolve, and this may severely limit the distance at which features on the flag could be identified, though without exhaustive information of Galileo's telescope the numerical impact is unknown.

Conclusion

We have calculated the maximum distances at which a telescope and the naked eye could resolve features in a ship's flag to be 35.5 km and 400 m respectively. Using a 17th century ship's speed, this leads to a 2.36 hour advantage, suggesting that Galileo's claim is accurate. We assumed that the lenses in the telescope are perfect, however Galileo's telescope suffered from spherical aberrations. This would have a significant impact on the resolving distance of the telescope and should be investigated in future research.

References

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