Journal of Physics Special Topics

An undergraduate physics journal

P1_2 Using Trebuchets for Lunar Satellites

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October 23, 2019

Abstract

In this paper we investigated the possibility of launching a 10 kg satellite into lunar orbit from the surface of the Moon, using a trebuchet. We found that the mass of the counterweight required would be 310 tonnes. We also discussed how the kinetic energy required for the speed of the orbit and the gravitational energy needed to be overcome changed as a result of the orbital radius.

Introduction

If we one day have a base or colony on the Moon, they might have a need to launch satellites into lunar orbit from the surface. Here we consider using a trebuchet to launch a small (10 kg) satellite into orbit and the mass of the counterweight needed to do so. We will need to consider the speed that the satellite would need to be travelling at to maintain its orbit and also the change in altitude that must take place.

The Trebuchet

The trebuchet is a medieval siege weapon that uses a counterweight to launch heavy objects (usually rocks) large distances. The mass of the counterweight affects the distance, as does the gravitational acceleration. If we ignore any energy lost from efficiency and specifics on rotational energy, then we can calculate the mass of counterweight required to give a satellite enough energy to be launched to an orbital altitude at orbital speed around the moon.

Method

Firstly, we must calculate the speed that the satellite must be travelling to be in orbit at our chosen orbital altitude of 100 km. We can calculate this using Equation (1),

$$v_s = \sqrt{\frac{GM}{r}}, \tag{1}$$

where v_s is the orbital speed of the satellite, G is the gravitational constant, M is the mass of the Moon and r is the orbital radius. Then, we use Equation (2) to calculate the kinetic energy of the satellite were it travelling at that speed,

$$E_k = \frac{1}{2} m {v_s}^2, \tag{2}$$

where m is the mass of the satellite. With that, we must then calculate the energy that must be given to the satellite to reach orbital altitude. Normally this would be calculated using the equation for the change in gravitational potential energy, however we must adapt it as it assumes that acceleration due to gravity is constant and we cannot make that assumption. Equation (3) combines $\Delta E_g = mg\Delta h$ and $g = \frac{GM}{r^2}$, with the integral for the changing acceleration due to gravity.

$$E_p = \int m \frac{GM}{r^2} dr, \tag{3}$$

where the Δh from the gravitational potential energy equation becomes dr. Then, we sum the two energies to find out the total energy that the trebuchet must transfer to the satellite assuming 100% efficiency and no atmospheric drag effects. From there we can then employ the gravitational potential energy equation to calculate the mass (Equation(4)) of counterweight required to transfer enough kinetic energy to the satellite.

$$m = \frac{\Delta E_t}{g \Delta h_c}, \tag{4}$$

where E_t is the sum of the kinetic and gravitational energies and Δh_c is the change in height of the counterweight.

Results and Discussion

For our calculations, we have assumed an orbital altitude of 100km. The mass of the Moon is 7.3×10^{22}kg [1] and the radius is 1.7×10^6m [1]. The acceleration due to gravity on the surface of the Moon is 1.6 m/s^2 [2]. The distance the counterweight moves is assumed to move through a height of 30m.

Using Equation (1) we calculate the orbital speed to be 1.6×10^3 m/s, the kinetic energy of that being 1.4×10^7 J. The change in gravitational potential energy (Equation (3)) - between the limits of 1.8×10^6 m (orbital radius) and 1.7×10^6 m (surface radius) - is calculated to be 1.6×10^6 J. The total energy is then 1.5×10^7 J.

Using this energy and Equation (4) we then find the required mass of counterweight to be 310 tonnes. This discounts any rotational effects on the counterweight and a 100% energy transfer from the counterweight to the satellite. If we were to use a heavier counterweight the satellite would be able to reach a higher orbit, however there are likely practical issues with such a large mass.

Comparing the kinetic and gravitational energies, we can see that - for our calculation - the kinetic energy is higher than the gravitational, however, this is not always the case. In Figure 1, we have made a graph to show how both the kinetic and gravitational energies change with the radius of the orbit desired. The total energy is also shown. The total energy curve seems to asymptotically approach the escape energy for the satellite.

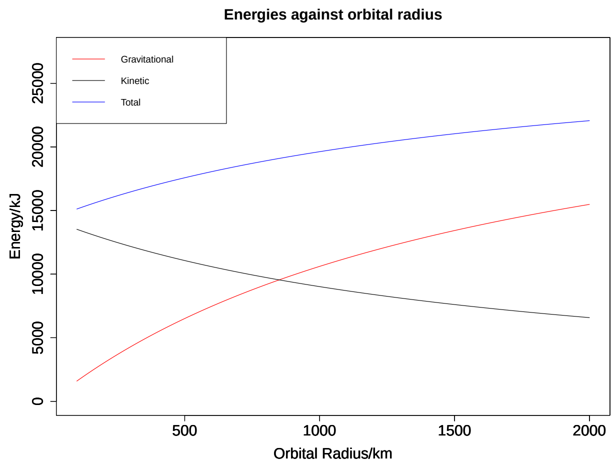

Figure 1: Shows the graph of the different energies. The gravitational energy follows a -1/r relation and the kinetic follows a 1/r relation.

Conclusion

Our calculations show that the mass of the counterweight must be 310 tonnes to launch a 10kg satellite into lunar orbit using a trebuchet. A comparison of the kinetic energy and gravitational energy showed us that - though it was lower for our calculation - there comes a point where the gravitational energy that needs to be overcome exceeds the kinetic energy required for the orbit. It was also illustrated that the total energy asymptotically approaches the escape energy for the satellite.

References

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[2] Williams, Dr. David R. (2 February 2006). "Moon Fact Sheet". NASA/National Space Science Data Center. Archived from the original on 23 March 2010. Retrieved 31 December 2008.