Journal of Physics Special Topics
An undergraduate physics journal

P5_2 A Mechanical Bee Analogue: Modelling the Elastic Pad Connecting a Bees Wing to Muscle as a Spring

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November 29, 2019

Abstract

We aimed to test the viability of modelling the elastic pad connecting a bee's wing to driving muscles as a spring and it was found this model works. It was found that the kinetic energy of the wings upstroke and the potential energy stored in the elastic pad were both around 40 erg, so the pad could be model as a spring with spring constant 360 N/m.

Introduction

A Bee flies by oscillating muscles in its thorax connected to wings through an elastic pad. The elastic pads stretch when the wings upstroke, storing the wings kinetic energy as potential energy, similar to a spring being stretched [1]. We investigated whether a spring could model this elastic pad.

Theory and Results

The elastic pad between the wing and thorax muscle of a bee acts like a spring, as it converts some of the kinetic energy of the upstroke into potential energy stored in the pad [1].

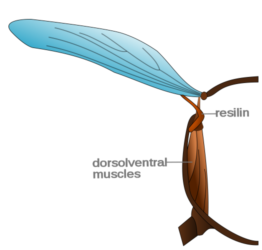

Figure 1: A bees wing connect to dorsolventral muscles through an elastic pad made of resilin [2].

To test the viability of using a spring as a model for the elastic pads, we must see if the energy stored within the pads when stretched is comparable to the kinetic energy produced by the wings upstroke.

The kinetic energy of the wings can be calculated by considering them in rotary motion:

$$KE = \frac{1}{2} I \omega_{\max}^2 \tag{1}$$

where I is the moment of inertia of a wing and ω_{max} is the maximum angular velocity during the upstroke. We approximated the wing as a thin rod pivoted at one end, with a moment of inertia I_{wing} of:

$$I_{wing} = \frac{m\ell^2}{3} \tag{2}$$

Where ℓ is the length of the wing, around 1 cm, and m is the mass of a wing is around 5x10^{-4} g [3]. The maximum angular velocity ω_{max} is found from the maximum linear velocity, ν_{max} at the centre of the wing:

$$\omega_{\max} = \frac{\nu_{\max}}{\ell/2} \tag{3}$$

Assuming the velocity varies sinusoidally along the upstroke, ν_{max} is twice the average velocity. During each upstroke the centre of the wings moves with an average velocity ν_{av} given by the distance moved by the centre of the wing divided by the duration of the upstroke. We can take the distance as 0.57 cm and the duration as 4.5x10^{3} s from experimental studies [3], giving ν_{av} as 127 cm/s. Substituting this into (3) we find ω_{max} = 50800 s^{-1} and thus using (1) we find the kinetic energy of the upstroke of both wings to be 43 erg.

Now we found the potential energy stored in the stretched pad. We assume the pad is a straight rod of area A and length ℓ and throughout the stretch the pad obeys Hooke's law. The volume of a resilin pad is taken roughly as a cylinder 210^{-2} cm long and 410^{4} cm^{2} in area and we assume the resilin rod is stretched by 50% of its original length [3]. The potential energy PE stored in the stretched pad is then:

$$PE = \frac{1}{2}\frac{YA\Delta\ell^2}{\ell} \tag{4}$$

Here Y is the Young's modulus for resilin, which is approximately 1.810^{7} dyn/cm^{2} [3]. Therefore, the potential energy stored in the pads is 36 erg.

This is comparable to the kinetic energy of the wings upstrokes found above meaning the pad can be approximated to a spring obeying Hooke's law.

Taking the energy stored in the pad above, we can calculate the spring constant of a spring in place of the pad using:

$$PE = \frac{1}{2}k\Delta\ell^2, \tag{5}$$

where k is the spring constant, which was found to be 360 N/m.

Discussion

We found the kinetic energy of the upstroke to be around 43 ergs, which is comparable to the potential energy of stretching the pad at 36 erg. However, this would be near 100% energy retention which is not realistic. This may be due to the assumption of the kinetic and potential energy: A bee's wing is larger and flatter than a rod, so the moment of inertia would be larger. Furthermore, the wing stroke path is not sinusoidal, being faster at the bottom, meaning the maximum velocity and angular velocity would be higher in reality, and therefor the kinetic energy would be greater than calculated. For the potential energy, the pad is actually a more complex shape than a cylinder, and as it stretches so far would change in diameter and young's modulus. This means it does not follow Hooke's law, so the potential energy would likely be much lower in line with experimental studies showing 80% energy retention [3]. The spring constant of 360 N/m is realistic for a spring [4], so one could be made as an analogue to the pad of a bee.

Conclusion

We found the energy stored in the elastic pads connecting a bee's wings and driving muscles to be of the same magnitude as the kinetic energy of the bee wings upstroke. Based on this we can take a spring as a good approximation to the elastic pad.

References

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[4] `https://us.misumi-ec.com/vona2/detail/110302288760/`[Accessed 3 October 2019]