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A4_1 Atoms Cubed!

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Abstract

The Rubik's Cube is famous for having a large number of permutations to it. We identify how large a Rubik's Cube would need to be for the number of combinations it has to exactly match the number of atoms in it for various dimensions of the cubes. We found that the $3 \times 3 \times 3$ Rubik's Cube, which normally has a side length of 5.7 cm $\pm$ 1 mm, is the only Cube that would meet this property at the visible scale while still being able to be held. When this Cube fits this given criteria it has a side length of 2.43 mm.

Introduction

The Rubik's Cube was invented in 1974 [1] and reached its peak in popularity in the 1980s [2]. Since then mathematicians have quantified the possible permutations, combinations and orientations that the Cube could have. We will determine how large a Rubik's Cube would need to be for its number of permutations to match the number of atoms in the Cube. We define a *permutation* as, "the position of pieces in the cube matrix", *orientation* as, "which way an element is facing", and *combination* as "a combination of permutations and orientation".

Number of Combinations

The formula for determining the number of combinations for a n×n×n Rubik's Cube from [3] is:

$$F(n) = \frac{\left(24 \times 12! \times 2^{10}\right)^{mod(n,2)} \times 7! \times 3^6 \times 24!^{\frac{n^2-2n}{4}}}{4!^{\frac{6(n-2)^2}{4}}} \quad (1)$$

In equation (1): n is the number of cubelets (a smaller cube of which the entire cube is made from) per row or column and $F(n)$ is the number of combinations.
The modulo function ("$mod(n,2)$") ensures that both even and odd (cubelet number) Rubik's Cubes follow the formula, which changes the number of combinations that a Rubik's Cube can have. From equation (1) it can be seen that the number of combinations increases dramatically with the number of cubelets. This is due to the high number of factorial functions that occur in this equation. For this reason we keep to cubelet sizes less than 10.

Assumptions

We will always refer to the original *Rubik's Cube* ®, and assume it is completely made from one of its thermoplastics, specifically acrylonitrile butadiene styrene (ABS)[4]. This will simplify the true composition, which is unknown to us. As the volume of the inner mechanisms are unknown to us, and the space between the mechanisms is minimal, we assumed a solid cube. We also assume that the cubelet size is consistent, and that the Rubik's cube remains a cube for all n.

Determining Number of Atoms

The number of atoms in the cube can be found using:

$$Atoms = \frac{N_a \times \rho \times V}{m_r} \quad (2)$$

Where N_a is Avogadro's constant, ρ is the density of ABS, V is the volume of the cube and m_r is the molecular weight of ABS.
ABS has a density of 1010-1080 kg m^{-3} [5] (we shall use slightly less than the maximum-1070 kg m^{-3}). Its chemical formula is $(C_8H_8C_4H_6C_3H_3N)_n$ and therefore has a molecular weight of 211.3022 g mol^{-1}. We measured a standard Rubik's Cube to have a side length of 5.7 cm ± 1 mm. Rearranging equation (2) for the side length of the cube gives us:

$$Length = \sqrt[3]{\frac{Atoms \times m_r}{N_a \times \rho}} \quad (3)$$

Results

Using equation (2) we can find out the number of atoms in the Rubik's Cubes of various cubelets. If we then set the number of combinations to be the number of atoms needed (in equation (3))we get the side length of the Cube.

N^0 of Cubelets	N^0 of Combinations	Side Length (m)
1	1.0000E+0	6.9175E-10
2	3.6742E+6	1.0674E-7
3	4.3252E+19	2.4282E-3
4	7.4012E+45	1.3481E+6
5	2.8287E+74	4.5409E+15
6	1.5715E+116	3.7330E+29
7	1.9501E+160	1.8620E+44
8	3.5174E+217	2.2665E+63
9	1.4170E+277	1.6739E+83

Table 1: Number of combinations and side length needed to have same number of atoms as combinations for a n×n×n Rubik's Cubes to 5 s.f.

Discussion & Conclusion

Table (1) shows that the length of the Cube dramatically increases with the number of cubelets. At a number of 6 cubelets, the side length (3.7330E+29 m) is greater than the size of the observable universe [6]. We believe that the *most* practical Rubik's Cube that could have the number of atoms matching the number of cubelets is 3 (side length of 2.43 mm), which just so happens to be the standard cubelet configuration.

References

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