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P4_8 Ringworld Dynamics

G. Thomas, D. King, R. Williams, Y. Abbas

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

In this paper, the dynamic properties of the ringworld structures in the video game series 'Halo' are investigated. The ringworlds rotate, and so have an associated centripetal force, which is approximately equal to the gravitational force of Earth [1]. The work done to accelerate each ring from rest, to an angular frequency which results in this centripetal force, was calculated to be 4.2×10^{24} kg. The angular frequency of each ring was calculated to be 1.4×10^{-3} rad s^{-1}. It was assumed that each ringworld was accelerated by a propulsion system, which ejects matter at high velocities, therefore producing thrust like a rocket engine or an ion propulsion system. To minimize the mass required to do this, the speed of the material was maximized. Therefore the speed was assumed to be equal to c, the speed of light in a vacuum. The mass of the material was calculated to be 4.0×10^{12} kg. It was assumed that each ringworld is identical.

Introduction

The video game "Halo: Combat Evolved" features a giant ring structure, built by an alien race known as the 'Forerunners', which exists in an orbit around the fictional gas giant 'Threshold' [1]. On the ring, there is a centripetal force, roughly equal to Earth's gravitational force [1], which acts on any mass on its surface.

In Kevin Grazier's "Halo Science 101", certain dynamical features of the ring were estimated, such as the total mass and radius. Using these physical properties, the energy required for the Forerunners to accelerate the ring to its necessary angular frequency was calculated. The ring was assumed to start at rest. The methods of accelerating the ring can only be guessed at, but a practical way of doing this is with a propulsion system. A pair of propulsion devices can be used to make a couple, which acts on the ring. A couple provides a torque with no net force, so the ring rotates without gaining translational speed. Each propulsion device provides angular momentum to the ring by ejecting mass continuously in opposite directions, along a line tangential to the surface.

In the calculation of the work done, the ring was assumed to have a thickness which is much smaller than the ring radius. It is reasonable to assume that the Forerunners would have wanted to minimize the material necessary to accelerate the ring to the required speed. It is also assumed that they had the technology to eject this material at extremely high speeds, i.e. very close to c, the speed of light in a vacuum. The ejected material has a linear momentum, which acts tangentially to the edge of the ring. This is directly proportional to the tangential speed and mass of the material. To minimize the mass, the velocity is therefore maximized. By assuming that the material is ejected at c, the minimum theoretical mass of ejected material was found. The calculation gives a slight under-approximation of the ejected mass, because nothing can travel at the speed of light according to relativity. It was also assumed that Threshold's gravitational field does not affect the results.

Theory

The work done, W, in rotating the ring from rest is equal to its final rotational kinetic energy, K, i.e. [2]

$$W = K = \frac{I_r \omega^2}{2} \tag{1}$$

Where I_r is the moment of inertia of a ring and ω is the angular frequency of the ring. To find the angular frequency, the centripetal force, F_c, on the ring and the force of gravitation on Earth, F_g, were considered and are given respectively below [2].

$$F_c = m\omega^2 r \tag{2}$$

$$F_g = \frac{GM_E m}{{R_E}^2} \tag{3}$$

Where m is an arbitrary mass which experiences either force, r is the ring radius, G is the gravitational constant, M_E is the mass of the Earth and R_E is the radius of the Earth. Equating the above and rearranging for ω gives,

$$\omega = \sqrt{\frac{GM_E}{{R_E}^2 r}} \tag{4}$$

The above is substituted into (1) for the calculation of work. The values of G, M_E and R_E used in the above calculation were, 6.67×10^{-11} $\text{Nkg}^{-2}\text{m}^2$, 5.97×10^{24} kg and 6.371×10^6 m respectively [2]. The value used for r was 5.0×10^6 m [1]. The form of the moment of inertia used in this calculation is that of a ring [2].

$$I_r = m_r r^2 \tag{5}$$

Where, m_r is the mass of the ring. The mass required to accelerate the ring to its necessary speed can be found from the ideal rocket equation [3].

$$v = c \ln \frac{m_o}{m_r} \tag{6}$$

Where, m_o is the mass of the ring and fuel combined and c is the speed of ejected fuel. The mass of the fuel, m_f, is given by,

$$m_f = m_o - m_r \tag{7}$$

Substituting this into the rocket equation and solving for m_f gives us

$$m_f = m_r \left(exp \left(\frac{\omega r}{c} \right) - 1 \right) \tag{8}$$

Where the mass of the ring, m_r, is 1.7×10^{17} kg [1]. The value used for c in the calculation for m_f was 3.0×10^8 ms^{-1} [2].

Results

The calculated value for the angular frequency was 1.4×10^{-3} rad s^{-1}. The work done on the ring was calculated to be 4.2×10^{24} J. The minimum possible mass of the ejected material was calculated to be 4.0×10^{12} kg.

Conclusion

The Forerunners would need a very large amount of material to accelerate the ring, but this could be obtained from the gas giant. The mass of "Threshold" is exactly half that of Jupiter [1], which has a mass in the order of 10^{27} kg [4], therefore there is enough material available to fuel the propulsion system. The work done is on the same order of magnitude as the energy incident on the Earth's surface from the Sun each year [5].

References

[1] Kevin Grazier (2007) "Halo Science 101".

[2] P.A. Tipler "Physics for Scientists and Engineers" sixth edition. Chapters 9-11.

[3] https://www.grc.nasa.gov/www/k-12/rocket/rktpow.html [Accessed 22/11/18]

[4] https://ssd.jpl.nasa.gov/?constants [Accessed 22/11/18]

[5] https://en.wikipedia.org/wiki/Solar-energy [Accessed 22/11/18]