Journal of Physics Special Topics
An undergraduate physics journal

P6_10 Satellite Five

K. Dhaliwal, G. Bower, K. Jiggens, P. Rojas Guijosa and J. Antonelli

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

November 23, 2018

Abstract

In the BBC show Doctor Who, the Doctor takes his companion Rose to a 500 floor broadcasting satellite named Satellite Five. The heat produced by the first 499 floors, is recirculated downwards in order to provide a cool environment for the alien editor on the 500th floor. We consider the effect of this on the external aluminium shell of the satellite. Heat produced by the 499 floors is found to be 4.25×10^{11} J. As a consequence, the temperature rise and volume expansion of the aluminium shell are found to be 16 K and 13 m^3 respectively. The pressure, if thermal expansion were to be restricted, is found to be 8.2×10^7 Nm^{-2}, which isn't enough to break the shell.

Introduction

'The Long Game' is a Doctor Who episode that focuses around the Doctor and his companion Rose visiting a 500 floor broadcasting satellite in orbit around the Earth, called Satellite Five. Satellite Five is designed to provide a cool environment for the alien editor at floor 500 so heat from the 499 floors below is recirculated. In this article, we investigate the amount of heat produced by 499 floors of the satellite and the corresponding rise in temperature over 24 hours. This would cause the volume of the external shell to expand and deform from it's natural state. However if no volume expansion could occur then pressure would build up. We then investigate whether or not this pressure increase would lead to the shell breaking.

Theory

To find the heat produced from 499 floors of the satellite, we examine the heat output of one average office on one floor, and extrapolate this to 499 floors. The heat given off by one human per day is 8.40×10^6 J [1]. The average heat given off one inefficient laptop per second is 100 J [2], this is 8.64×10^6 J per day. Assuming 50 human beings on one floor, 50 laptops and no other sources of heat, the average heat given off on one floor is 8.52×10^8 J. Multiplying this result by 499 gives the heat produced by 499 floors to be 4.25×10^{11} J. We have assumed a 24 hour working day and that the satellite's shell is made of aluminium [3]. We also assume all the heat is absorbed by the walls and it doesn't radiate to the surroundings. This would lead to continuous increase in temperature, hence we have only considered a 24 hour period.

We approximate the main body of Satellite Five to be a cuboid, similar to the New York Times building, and take the thickness of a wall at 0.09 m (3.5 inches) [4]. The New York Times building has 52 floors and stands at a height of 220 m [5], so 1 floor has a height of 4.23 m. Satellite Five has 500 floors so it's height is 2120 m We assume one floor of Satellite Five has a cross sectional area as that of a large square boardroom, for 50 people, at 187 m^2 [6], adding on the thickness of the walls gives an area of 192 m^2. The total external volume, V_{tot}, is found to be 4.07×10^5 m^3.

However, we are considering the volume expansion of the aluminium shell of Satellite Five, so to find the volume of the shell, V_{shell}, we need to subtract the volume of the interior, V_{int}, from V_{tot}. The interior height is 2120 m, multiplying this by the internal floor area (187 m^2 [6]) gives the internal volume of 3.96×10^5 m^3. Subtracting this from V_{tot}, gives V_{shell} at 1.10×10^4 m^3.

By using V_{shell} and the density of aluminium, which is 2.7×10^3 kg m^{-3} [7], we find the mass of the shell to be 3.0×10^7 kg.

We can use the equation below to find the temperature change:

$$\Delta T = \frac{Q}{mc} \tag{1}$$

Where Q is the heat produced by the 499 floors (4.25×10^{11} J), m is the mass of the shell (3.0×10^7 kg), c is the specific heat capacity of aluminium (900 Jkg^{-1} K^{-1}) [8] and ΔT is the temperature change. ΔT is found to be 16 K.

Volume expansion can be found using:

$$\Delta V = \beta V_0 \Delta T \tag{2}$$

Where ΔV is the change in volume, β is the coefficient of volume expansion for aluminium (7.2×10^{-5} K^{-1}) [9], V_0 is the initial volume (i.e. V_{shell}) and ΔT is the temperature change (16 K). We find the volume change to be 13 m^3.

If Satellite Five is designed to not permit thermal expansion of the aluminium shell, the resulting build up of pressure can be found using:

$$P = B\frac{\Delta V}{V_0} \tag{3}$$

Where P is the pressure, B is the bulk modulus of aluminium. Taking B to be 6.9×10^{10} Nm^{-2} [10] and substituting in for ΔV and V_0, we find the pressure to be 8.2×10^7 Nm^{-2}.

Conclusion

We find the overall temperature change would lead to a large volume expansion of the shell. The strongest aluminium alloy can take a pressure of 5.0×10^8 Pa (7.2×10^4 psi) [11]. If volume expansion was restricted, assuming it's made from the same alloy, then we find the aluminium shell will not break despite the large pressure build up.

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