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# P4 5 Falling Into Mount Doom 

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#### Abstract

During a scene in the film "The Lord of the Rings: Return of the King", a creature called Gollum falls into a volcano (Mount Doom) and sinks into the lava. In this article we investigate the displacement of Gollum once he enters the lava inside Mount Doom. We assumed that Gollum enters the lava at terminal velocity, which we calculated to be $33 \mathrm{~m} \mathrm{~s}^{-1}$. We found that Gollum would sink to 0.15 m below the surface of the lava but would then rise to the surface 0.27 s after hitting the lava. Therefore, we believe that the physics shown in the film is incorrect.


## Introduction

When an object falls onto a surface, some of its kinetic energy is converted into sound energy as well as into the kinetic energy of any matter displaced due to the impact. However, for simplicity we are going to assume that the kinetic energy lost due to the collision with the surface is negligible, therefore Gollum will have an initial velocity in the lava equal to his terminal velocity in the air. We are also going to assume that he is fully submerged as soon as he hits the lava. Once in the lava the Gollum will experience three forces: the drag force $\left(F_{d}\right)$ caused by his motion through the lava, the buoyancy force $\left(F_{b}\right)$ acting upwards on him and the weight $\left(F_{w}\right)$ of Gollum acting downwards. Throughout, we will ignore the effects the high temperatures will have on Gollum's body

## Theory

The terminal velocity of Gollum through the air is given by [1]:

$$
\begin{equation*}
v=\sqrt{\frac{2 m g}{\rho_{a} A C}} \tag{1}
\end{equation*}
$$

Where $m$ is Gollum's mass which we assumed to be $20 \mathrm{~kg}, g$ is the gravitational field strength $\left(9.81 \mathrm{~m} \mathrm{~s}^{-2}\right), \rho_{a}$ is the density of air $(1.225 \mathrm{~kg}$ $\left.\mathrm{m}^{-3}[2]\right), A$ is the cross-sectional area of Gollum and $C$ is Gollum's drag coefficient. The cross-sectional area for a human is $0.7 \mathrm{~m}^{2}$ [1], but Gollum is less than half the size of the average human so we set $A$ to $0.3 \mathrm{~m}^{2}$. We modelled Gollum as a humanoid in a "belly to Earth position", therefore he has a drag coefficient $C$ of 1 [1]. Substituting these values into Equation (1) gives a terminal velocity of $33 \mathrm{~m} \mathrm{~s}^{-1}$.
The buoyancy force is equal to the weight of the displaced fluid, in this case the weight of the lava displaced by Gollum. First we will consider his deceleration when travelling downwards through the lava. $F_{w}$ acts downwards, whereas $F_{d}[3]$ and $F_{b}$ act upwards, this means that the total force $(F)$ acting on Gollum is:

$$
\begin{equation*}
F=F_{d}+F_{b}-F_{w}=\frac{1}{2} C \rho_{l} A v^{2}+m_{d l} g-m g \tag{2}
\end{equation*}
$$

Where $\rho_{l}$ is the density of lava ( $3100 \mathrm{~kg} \mathrm{~m}^{-3}$ [4]), $v$ is Gollum's velocity and $m_{d l}$ is mass of
the displaced lava. The mass of the displaced lava and the mass of Gollum whose volume is $V$ can be written as $\rho_{l} V$ and $\rho_{g} V$ respectively, where $\rho_{g}$ is Gollum's density. Therefore:

$$
\begin{equation*}
F=\frac{1}{2} C \rho_{l} A v^{2}+\left(\rho_{l}-\rho_{g}\right) V g \tag{3}
\end{equation*}
$$

Using Newton's 2nd law, dividing Equation (3) by Gollum's mass $m$ gives his deceleration $a$.

$$
\begin{equation*}
a=\frac{\frac{1}{2} C \rho_{l} A v^{2}+\left(\rho_{l}-\rho_{g}\right) V g}{m} \tag{4}
\end{equation*}
$$

Table 1: Values for Equation (4).

| C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | | $\rho_{l}$ |
| :---: |
| $\mathrm{~kg} \mathrm{~m}^{-3}$ | | A |
| :---: |
| $\mathrm{m}^{2}$ | | $\rho_{g}$ |
| :---: |
| $\mathrm{~kg} \mathrm{~m}^{-3}$ | | V |
| :---: |
| $\mathrm{m}^{3}$ | | g |
| :---: |
| $\mathrm{m} \mathrm{s}^{-2}$ | | m |
| :---: |
| kg |

As Gollum is very skinny we assumed that his density is equal to the average lean density of a human, $1096 \mathrm{~kg} \mathrm{~m}^{-3}$ [5]. Based on his size, Gollum's mass $m$ was assumed to be 20 kg , from this we calculated his volume $V$ to be $0.02 \mathrm{~m}^{3}$.

Equation (4) shows that the deceleration is not uniform as $a$ depends on $v$ which changes with time. Therefore to calculate the distance Gollum travels, we approximated the situation as a series of very small uniform accelerations.

Substituting the values from Table 1 and the initial velocity calculated earlier ( $33 \mathrm{~m} \mathrm{~s}^{-1}$ ) into Equation (4) gives a deceleration of $25339 \mathrm{~m} \mathrm{~s}^{-2}$. We then used $v=u+a t$, where $u$ is the initial velocity and $t$ is the time ( 0.00001 s ) to calculate a new velocity of $32.75 \mathrm{~m} \mathrm{~s}^{-1}$. Finally we used $x=0.5(u+v) t$ to calculate a displacement $x$ of $3.29 \times 10^{-4} \mathrm{~m}$. The value of $t=0.00001 \mathrm{~s}$ was chosen as during this short time period the acceleration will not change significantly.

We then repeated the process every 0.00001 s , thousands of times and found that after 0.072 s Gollum's velocity reaches 0 , this corresponds to a total displacement of 0.15 m below the lava surface. Gollum now starts moving back up towards the surface, so the drag force $F_{d}$ acts in the opposite direction, therefore Equation (4) becomes:

$$
\begin{equation*}
a=\frac{-\frac{1}{2} C \rho_{l} A v^{2}+\left(\rho_{l}-\rho\right) V g}{m} \tag{5}
\end{equation*}
$$

Using a similar process as before but starting by substituting $v=0$ into Equation (5) we found it would take another 0.20 s for Gollum to get to the surface again. Therefore, Gollum would spend 0.27 s under the lava.


Figure 1: Graph showing the displacement of Gollum below the surface as a function of time.

## Conclusion

We believe that the physics shown in this scene from is wrong, as we have shown Gollum would float on the lava as he is less dense than lava, plus he will only be under lava for 0.27 s . During this time we will reach a maximum depth of 0.15 m .

## References

[1] https://en.wikipedia.org/wiki/Speed_ skydiving [Accessed 12/11/18]
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[4] https://perryponders.com/2017/04/04/ you-would-not-sink-in-lava/ [Accessed 12/11/18]
[5] S. B. Heymsfield et al, Chemical determination of human body density in vivo: relevance to hydrodensitometry, The American Journal of Clinical Nutrition, Volume 50, Issue 6, 1 December 1989, Pages 1282-1289.

