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# A6_5 Floating Cities

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**Abstract**

In this paper we describe the construction of a floating habitat as described by Buckminster Fuller in the 1930's. We first examine a 2 km wide sphere that has been heated by 1°C and find it generates a lift of $1.64 \times 10^8$ N. We further examine the feasibility of carbon nanotubes as a construction material and find that they are able to withstand the tension from the interior stresses of the sphere. Lastly, we consider the general case with variable radii and temperatures, and find that even the smaller spheres can house thousands of people.

## Introduction

In the 1930's Buckminster Fuller proposed creating habitats from geodesic spheres and levitating them by slightly heating the air inside. When he originally conceived of the idea it was only as a thought experiment to promote creativity. In this paper we will investigate the viability of such a construction project using the best available materials.

## Theory

In this paper we will start by considering the construction of a 'habitat sphere' 1 km in radius located in an area with temperature 15°C. From Archimedes' principle we know that the lift generated (buoyancy) by heating the air inside must equal the weight of air displaced. The change in air density, $\Delta\rho$, for a specific change in temperature ($T_1$ to $T_2$) is given by the ideal gas law;

$$\Delta\rho = \frac{p}{R_{specific}T_1} - \frac{p}{R_{specific}T_2}, \qquad (1)$$

where $p$ is the ambient pressure and $R_{specific}$ is the specific gas constant for dry air. Multiplying $\Delta\rho$ by the weight of air in the sphere gives a value for the lift generated. For a change in temperature of 1°C, a lift force of $1.64\times10^8$ N can be calculated. This is a relatively large force, and we would therefore require the strongest building materials to construct the sphere. We will consider carbon nanotubes in rods of cross-sectional area 1 $mm^2$. One particular nanotube, that we will use here, has a density of 1.3 $gcm^{-3}$ [1] and can take a maximum weight of 6422g N [2]. The hypothetical habitat will be constructed using meter long rods of mass $1.3\times10^{-3}$ kg connected into tessellating triangles of area 0.5 $m^2$. The triangles will be placed such that they form a geodesic sphere. As a result of the tessellation, each triangle only contains two unique rods, so the total number of rods is twice the number of triangles (plus one). In other words, to form a new triangle in the structure, only two new rods are required. To find the total number of triangles required we divide the surface area of the sphere by the area of an individual triangle. This means that a habitat of radius 1 km will take $5\times10^7$ rods and have a total mass $6.5\times10^4$ kg. We can observe that this mass (and resulting weight) is negligible compared to the lift generated. We propose the houses would sit on a thin sheet of negligible weight, wrapped around the outside of the shell. This sheet would also be sufficient to store the air required for heating. Another concern is whether or not the rods could take the strain caused by the lift. The tension would be greatest at the equator of the sphere which would contain roughly $\pi d = 6283$ rods. Assuming that the maximum strain is shared evenly between the rods, each would feel a force of 26102 N. Given that each rod can withstand 62999 N, we conclude that the structure is stable and able to lift itself.

### Analysis

When constructing these habitats there are two major parameters we can modify. One is the temperature difference from inside the habitat to outside and the other is the radius of the habitat. The temperature provides a 1/dT contribution to lift (see (1)), although the radial dependence is much more significant; varying by $r^3$ . This means that it is much more effective to vary the radius of the habitat, however this comes with increases to construction costs. In order to find the best way to house the most people we made a contour plot of the population that can be housed for different temperature and radii.

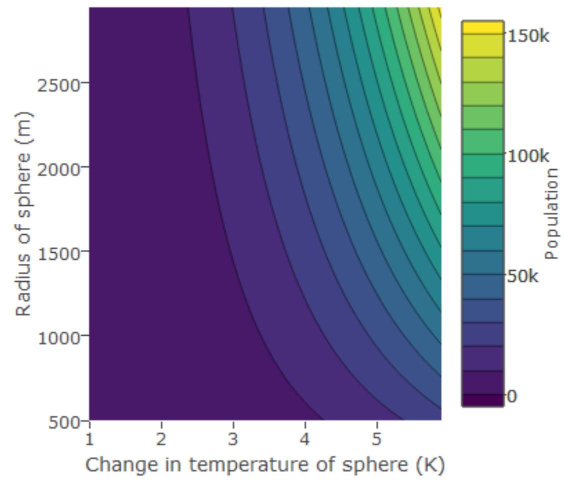


Figure 1: A contour plot for population given different radii and temperatures.

In order to generate this plot we have assumed that the average house weighs 50000 kg and holds three people. This graph shows us that if people can tolerate living at temperatures a few degrees higher than normal then even the smallest of habitats can hold many thousands of people.

### Conclusion

Using new production methods carbon nanotubes could be produced as cheaply as $15 per gram[3] this means that construction of a 1 km habitat frame would cost under 10 million dollars which is a relatively low cost to house thousands of people. Additionally, the cost to heat the air inside is just under 1 million dollars and because the temperature gradient between the inside and outside of the sphere is so small it will lose heat slowly so the cost of maintaining the temperature difference should be negligible. What might initially seem an outlandish and impractical thought experiment can possibly achieved for less than $15 million making it quite feasible as a new form of housing for countries faced with significant overpopulation or as emergency housing in areas prone to earthquakes and flooding.

### References


[1] Philip G Collins and Phaedon Avouris. Nanotubes for electronics. *Scientific american*, 283(6):62–69, 2000.

[2] Min-Feng Yu, Oleg Lourie, Mark J Dyer, Katerina Moloni, Thomas F Kelly, and Rodney S Ruoff. Strength and breaking mechanism of multiwalled carbon nanotubes under tensile load. *Science*, 287(5453):637–640, 2000.

[3] New method for continuous production of carbon nanotubes. `https://www.sciencedaily.com/releases/2012/04/120412105109.htm`. Accessed:22,10,2018.