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P1_4 Starship UK, the Hardest of Brexits?

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Abstract

In this paper we estimate the amount of energy and mass of fuel required to move the landmass of the United Kingdom, including the crust down to a depth of 24 km, into a Low Earth Orbit. We also compared launch vehicles in terms of how much they can carry to orbit, and thus how many would be required and how much this would cost. It was found that 5.17×10^{26} J of energy would be required, or 1.15×10^{14} Saturn V launchers at a cost of £1.61×10^{22}.

Introduction

The result of the June 2016 Referendum indicated that the British people wished to leave the European Union. We investigate a hypothetical situation in which the country decided to leave Earth entirely. A certain amount of energy, and thus a certain change in velocity, is required to lift a mass to orbit. This can be used to determine the mass of propellant required. Each of these were calculated in sequence using an estimate for the mass of the UK. We then compare launch vehicles to determine how many would be required to lift the UK.

Calculations

The surface area of the UK is known to be 2.43×10^{11} m^2 [1], and the depth of the Earth's crust beneath the UK varies between 24 and 36 km [2], so to minimise mass, save energy and avoid cutting into the mantle at any point, we use a value of 24 km for the thickness of the crust. Continental crust is composed primarily of granite [3], so we take the density of granite, (2750 kgm^{-3}) as the density of the UK and its crust [4]. Multiplying the surface area by the crustal depth gives a volume of 5.84×10^{15} m^3. To calculate the mass we then use

$$\rho = \frac{M}{V} \tag{1}$$

where ρ is the density of the UK, and M and V are the mass and volume respectively [5]. Rearranging for mass results in a value of 1.61×10^{19} kg. We shall assume a circular Low Earth Orbit (LEO). Normally, the minimum altitude for a stable LEO is 160 km [6], but we will need to orbit at at least 184 km to account for the 24 km depth of our orbiter and avoid heavy orbital decay. We then calculate the UK's orbital velocity and change in energy to reach this altitude using

$$v^2 = \frac{GM}{r} \tag{2}$$

$$\Delta E = \frac{GMm}{2r} + GMm\left(-\frac{1}{r} + \frac{1}{R}\right) \tag{3}$$

where v is the orbital velocity, G is the universal gravitational constant, m is the mass of the UK, M is the mass of the Earth, r is the orbital radius and R is the Earth's radius [7]. Using our values for mass and orbital radius, we find v to

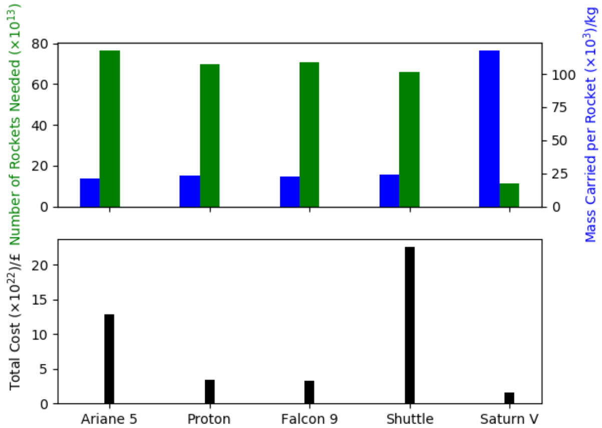

Figure 1: Two graphs showing the mass-to-orbit capabilities of various launchers, the number of them required to lift the UK's mass, and the total cost of buying that many.

be 7.79 kms^{-1} and ΔE to be 5.17×10^{26} J. The Tsiolkovsky Rocket Equation states that

$$\Delta v = v_e \ln\left(\frac{m_0}{m_f}\right) \tag{4}$$

where, v_e is the exhaust velocity, and m_0 and m_f are the initial and final masses of the rocket respectively [8], allowing us to estimate the propellant needed to achieve the calculated velocity. Using a value of v_e for liquid-fuelled rockets of 4.4 kms^{-1} [9] and assuming that the final mass is only that of the UK, we find m_0 to be at least 9.46×10^{19} kg, meaning that at least 7.85×10^{19} kg of propellant is needed to lift the UK, which is several times the country's mass.

Figure 1 is a comparison of various heavy lift launch vehicles, with the masses they are capable of transporting to orbit, and from that, the number of them that would be required to lift the UK, and the cost of enough launchers to do so [10] [11], neglecting the fact that the mass of the UK is all in one rather than manageable portions.

Selecting the Saturn V as our launch vehicle for this endeavour (it has the lowest total cost of £1.61×10^{22}), with each costing £141 million to launch [10], we would need 1.15×10^{14} of them, assuming enough Saturn V launchers existed and we could get them under the UK somehow.

Conclusion

We have found that it would take 5.17×10^{26} J of energy to launch the UK into a stable circular LEO. Doing this would require a minimum of 7.85×10^{19} kg of propellant and would cost £1.61×10^{22}. To further develop this paper, fewer simplifications and assumptions would be made, such as not neglecting air resistance and gravitational effects in our calculations.

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