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P6_6 Do You Want to Build a Snowman?

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Abstract

We investigate the temperature change that Elsa, from Disney's Frozen, undergoes as she uses her ice power. Our findings show that, if modelled as a typical refrigerator, she would endure a temperature increase far greater than a human could withstand. To create the snowman, Olaf, from water vapor in the air she would have a temperature change of 131 K. To freeze the city Arendelle, she would see an huge temperature rise of 3.6×10^{7} K.

Introduction

In Disney's Frozen, Elsa is seen creating a snowman, Olaf, out of the water vapor in the air along with freezing the city of Arendelle. We investigate the temperature change that she will experience when she accomplishes this. Arendelle is based on Arendal, a city in Norway [1], which has an area of 270 km^2 [2]. The film takes place in July [1], so the average outside temperature was taken to be 16 °C (289 K), corresponding to that in Arendal [3].

Theory

Modelling Elsa as a refrigerator, we can calculate W, the work done by her.

$$W = \frac{Q_c}{cop} \tag{1}$$

Where Q_c is the heat absorbed from the air and *cop* is the coefficient of performance of Elsa. A typical refrigerator *cop* has a value of 5.5 [4].

$$Q_c = Q_v + Q_f + Q_{cool}, \tag{2}$$

$$\begin{aligned} Q_v &= mL_v && (3)\\ Q_f &= mL_f && (4)\\ Q_{cool} &= mc_w\Delta T_1 && (5) \end{aligned}$$

where L_f and L_v are the latent heat of fusion and vaporisation which are 2.26 MJ kg^{-1} and 333.5 kJ kg^{-1}, respectively for water. c_w is the specific heat capacity of water; 4.18 kJ kg^{-1} K^{-1} [4]. m is the mass of the frozen water and ΔT_1 is the temperature change of the water, this is 16 K, which is the difference between the freezing point of water and the average temperature in Arendal in July.

Elsa's change in body temperature, ΔT_e, is related to the heat absorbed by Elsa, Q$_h$, by:

$$Q_h = m_e c_e \Delta T_e = W + Q_c \tag{6}$$

Where m_e is Elsa's mass, taken to be that of an average human, 70 kg [5] and c_e is the specific heat capacity of Elsa, 3.49 kJ kg^{-1} K^{-1} [6].

Results

Combining equations 1 - 6 and rearranging for the temperature change of Elsa gives:

$$\Delta T_e = m\frac{(L_v + L_f + c_w\Delta T_1)(1 + \frac{1}{cop})}{m_e c_e} \tag{7}$$

Plotting this gives figure 1.

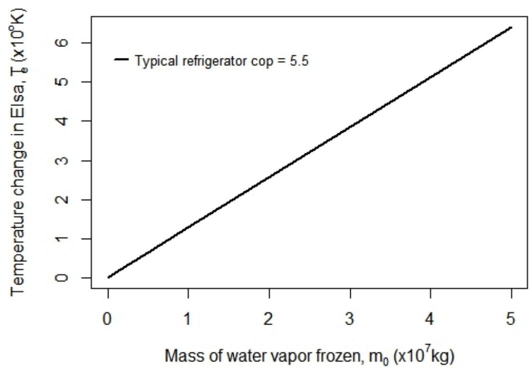

Figure 1: Temperature increase in Elsa as a function of the mass of water she is converting to ice.

Olaf is 2 ft 6 inches tall [7], assuming he is made up of three balls of snow with dimensions as given in table (1) he will have a total volume of 0.17 m^3. Using the equation for the volume of an ellipsoid we generate row 4 of table (1). Fresh snow has a density of approximately 60 kg m^{-3} [8]. This means Olaf has a mass of 10.2 kg. Using a water mass of 10.2 kg in Eq. (7) gives Elsa a temperature change of 131 K. Elsa also freezes the city. At 16 °C and relative humidity of 75% [9] there are 8.43 g of water per kg of air [10]. Given the density of air at sea level (1.25 kg m^{-3}) [11], there are 10.5×10^{-3} kg of water per m^3. Assuming Elsa only freezes the first meter of air above the ground, she freezes 2.8×10^6 kg of water. Using this water mass in Eq (7) Elsa would experience a temperature change of 3.6×10^7 K.

Conclusion

We have found that when modeling Elsa as a refrigerator, building Olaf and freezing Arendelle would result in a rise in temperature that no human could withstand. To further this study, Elsa could be modelled as a Carnot refrigerator to see if the increase in efficiency would be enough to make considerably lower Elsa's change in temperature.

	Head	Middle	Bottom
Height (m)	0.38	0.13	0.25
Width (m)	0.25	0.13	0.25
Depth (m)	0.25	0.13	0.25
Volume (m^3)	0.099	0.0092	0.065

Table 1: Olaf's dimensions.

References

[1] http://disney.wikia.com/wiki/Arendelle [Accessed 9 October 2018]

[2] https://en.wikipedia.org/wiki/Arendal [Accessed 9 October 2018]

[3] https://www.holiday-weather.com/arendal_no/averages/ [Accessed 9 October 2018]

[4] Tipler, P. and Mosca, G. (2008). Physics for scientists and engineers. New York: W.H. Freeman, Chapter 19.

[5] https://hypertextbook.com/facts/2003/AlexSchlessingerman.shtml [Accessed 11 October 2018]

[6] http://www.zuniv.net/physiology/book/chapter21.html [Accessed 9 October 2018]

[7] http://goldflshturtle.tumblr.com/post/84245299773/heights-of-frozen-characters-anna-50-152 [Accessed 9 October 2018]

[8] https://www.sciencelearn.org.nz/resources/1391-snow-and-ice-density [Accessed 9 October 2018]

[9] http://hikersbay.com/climate/july/norway/arendal?lang=en [Accessed 21 October 2018]

[10] https://www.lenntech.com/calculators/humidity/relative-humidity.htm7 [Accessed 21 October 2018]

[11] https://hypertextbook.com/facts/2000/RachelChu.shtml [Accessed 9 October 2018]