

A2_2 The Terrifying Trolley Trick

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Abstract

This paper explores the feasibility and practicality of being able to cross the Grand Canyon in the most obtainable mode of student transport, the shopping trolley. It was found that, given a simplified model, it is possible to travel 90.2 m in the trolley, travelling further than the 69 m completed by Robbie Knievel at the same point in May 1999.

Introduction

On 20th May 1999, Robbie Knievel jumped the Grand Canyon on a 500 cc motorcycle, reaching speeds of 80 mph (36 ms^{-1}) [1]. Using information about the Grand Canyon and the dynamical properties of a supermarket trolley, it is possible to see if students can perform a similar feat in a more daring yet practical fashion.

Theory

In order to determine the feasibility of such a stunt, two factors must be considered. First, the width of the Grand Canyon at the point where the jump would take place. This is set to be 69 m [1] in the same section where Robbie Knievel made the jump. Second, the maximum velocity achievable by a shopping trolley while travelling down a ramp due to gravity alone, which is governed by its terminal velocity. This is calculated by equating the force due to gravity (weight) to the drag forces on the trolley, which consist of the summed total of the air resistance and friction forces, as represented in Figure 1.

Here, mg represents the force due to gravity, where m is the total mass of the trolley and driver (kg), and g is the acceleration due to grav-

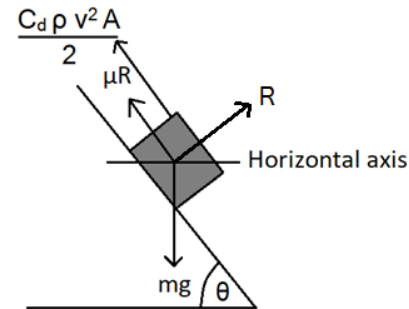


Figure 1: Diagram of the forces acting on the trolley in the direction of travel while accelerating on the ramp

ity at sea level (ms^{-2}). $\mu R = \mu mg \cos(\theta)$ represents the force due to the friction from the wheels on the surface, where μ is the coefficient of friction produced by the contact of the two materials, which can be estimated to be estimated to $\mu = 0.303$ for hard rubber wheels on steel [2], and R is the reaction force (N), which is dependent on the angle of the downward ramp from the horizontal, θ . $(C_d \rho v^2 A)/2$ represents the drag force due to the air resistance acting on the trolley [3], where C_d is the drag coefficient, ρ is the air density (kgm^{-3}), v is the velocity of the trolley

(ms^{-1}) and A is its frontal surface area in the direction of travel (assumed 1 m^2). For the sake of simplicity, the trolley is considered to be a cube of mass 90 kg of even distribution, which can be estimated to have a drag coefficient of 0.80 [4].

Hence, resultant equation for the balance of forces is produced:

$$mg\sin(\theta) = \mu mg\cos(\theta) + \frac{C_d\rho v^2 A}{2} \quad (1)$$

Rearranging for the terminal velocity gives:

$$v_{\text{term}} = \sqrt{\frac{2mg(\sin(\theta) - \mu\cos(\theta))}{C_d\rho A}} \quad (2)$$

It can now be seen that the terminal velocity is a function of θ , and all the other variables are kept constant. This can now be substituted into equation 3, where S is the range a projectile.

$$S = \frac{v^2 \sin(2\phi)}{g} \quad (3)$$

where ϕ is the angle of the launch ramp, to give:

$$S = \frac{2mg(\sin(\theta) - \mu\cos(\theta))}{C_d\rho A} \frac{\sin(2\phi)}{g} \quad (4)$$

(note that the air resistance acting on the trolley while airborne is assumed to be negligible due to the short amount of time while in the air). It can be seen that the range increases with increasing ϕ , up to $\phi = 45^\circ$, where $\sin(2\phi) = 1$. Hence, the launch ramp will be set to an angle of 45° from the horizontal, with assumed negligible energy loss. Also, S increases with increasing θ , up to $\theta = 90^\circ$. Any constraints can now be applied. While the trolley is travelling on the ramp, there is a point where it will tip over if the line of action of its weight falls outside its base. The maximum ramp angle can therefore be calculated by using equation 5 [5].

$$\theta_c = \arctan\left(\frac{t}{2h}\right) \quad (5)$$

Where θ_c is the critical angle of tipping (degrees), t is the width of the base (m) and h is

the height of the centre of mass (assumed to be 0.5 m).

Results

Using equation 2, the terminal velocity was calculated to be $v_{\text{term}} = 26.9 \text{ ms}^{-1}$. In order to calculate the maximum range, the critical angle is required (since the maximum range is a function of ϕ only). Therefore $\theta = \theta_c$ at this maximum. Substituting $t = 1 \text{ m}$ and $h = 0.5 \text{ m}$ into equation 5, a critical angle of $\phi = 45^\circ$. Using this result with equation 4 allows a value to be obtained for the range, $S = 90.2 \text{ m}$. Therefore according to the model used for this paper, it is theoretically possible to be able to cross the Grand Canyon using a shopping trolley.

Conclusion

This investigation has displayed that it would be possible to traverse the Grand Canyon in a supermarket trolley, a feat that students previously believed they could only dream of. There are additional factors to consider if investigating this further, for example: a more accurate model of the weight distribution to find the centre of mass, including air resistance during flight, calculating how long the slope would be to reach terminal velocity and the cost of such a slope.

References

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