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A6_4 The Hyperloop through the centre of the Earth

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Abstract

In this paper a description of how a Hyperloop style pod would travel through a low pressure tunnel straight through the Earth is provided. The equations of motion for the pod in an airless tube are derived, and the total travel time is found to be in agreement with the commonly quoted value of 42 minutes. A peak velocity of about 8000 ms^{-1} was also calculated. Furthermore, a numerical solution to an Earth-bisecting tube with low air resistance was found, and graphed to show an exponential decay in amplitude of trajectory. The loss of velocity in every oscillation meant the pod would not reach anywhere near the other side of the planet.

Introduction

Travelling down a tunnel through the centre of the Earth would undoubtedly be effective for intercontinental travel. If it was possible, it would cut the distance travelled significantly compared to moving across the surface. Using the parameters of a low pressure tube published by Elon Musk's company, 'Hyperloop', the kinematics of travel for vacuum and near-vacuum conditions through the planet are investigated. The Hyperloop concept works by utilising a very low air pressure ($\sim$100 Pa) to dramatically reduce air resistance faced by the pod, allowing it to accelerate to high speeds.

Analysis

To set up a differential equation for the pod's motion a suitable coordinate system must be defined. If y is the position of the pod along the horizontal line through the center of the Earth, and $y = 0$ is the centre of the planet, the forces on the pod can be expressed as a function of y. This gives Eq.(1);

$$\vec{F_{total}} = \vec{F_{gravity}} + \vec{F_{air\ resistance}}. \tag{1}$$

Note that although $\vec{F_{gravity}}$ is always pointed towards the centre of the planet, $\vec{F_{air\ resistance}}$ points the opposite direction to the velocity vector.

Firstly, the case without air resistance is considered and is represented as a differential equation below:

$$\vec{F_{total}} - \vec{F_{gravity}} = 0, \tag{2}$$
$$\Rightarrow \frac{d^2y}{dt^2} - \frac{4}{3}G\rho_{earth}\pi y = 0.$$

This equation has a general solution of:

$$y = A\cos(\alpha t) + B\sin(\alpha t). \tag{3}$$

Solving using the known boundary conditions: $y = R_E$, $\frac{dy}{dt} = 0$ and $\frac{d^2y}{dt^2} = -9.81$ ms^{-2}, it can be shown that:

$$y = R_E \cos\left(\sqrt{\frac{4}{3}\pi\rho G t}\right). \tag{4}$$

Differentiating this equation, the velocity is found to be:

$$v = -R_E\sqrt{\frac{4}{3}\pi\rho G}\sin\left(\sqrt{\frac{4}{3}\pi\rho G t}\right). \qquad (5)$$

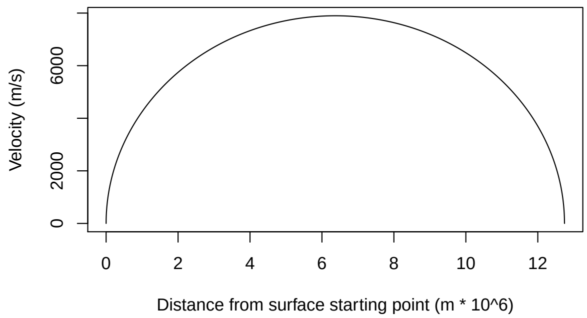

Figure 1: Plot of vactrain velocity as it travels through the Earth.

These equations provide the kinematics of the pod in a vacuum which have been plotted in Figure 1. The time of flight through the earth is then calculated as 2544 seconds, and the maximum velocity is found to be 7900 ms^{-1}. These results agree with the commonly quoted travel time of 42 minutes [1].

A more accurate model was then used to examine the kinematics of the pod through the tube with air resistance using the differential equation:

$$\frac{d^2y}{dt^2} + \frac{\rho_{air}C_D A}{2m_{pod}}\left(\frac{dy}{dt}\right)^2 - \frac{4}{3}G\rho_{earth}\pi y = 0, \quad (6)$$

where C_D is the drag coefficient for dry air, modelled as 0.04 for a streamlined body. A is the cross-sectional area of the pod, cited as 3.9 m^2, and ρ_{air} is the density of air in the tube; given as 0.1% of the air density on Earth [2]. This comes out to be 1.2×10^{-3} kgm^{-3}. The mass of the pod, m_{pod}, is assumed to be 500 kg. Numerical methods were applied to this second order differential equation as no analytical solution could be found. This solution is plotted in Figure 2 using the same boundary conditions as the first case. The solution can be described as an exponential decay (in amplitude) as the pod oscillates about the centre of the Earth.

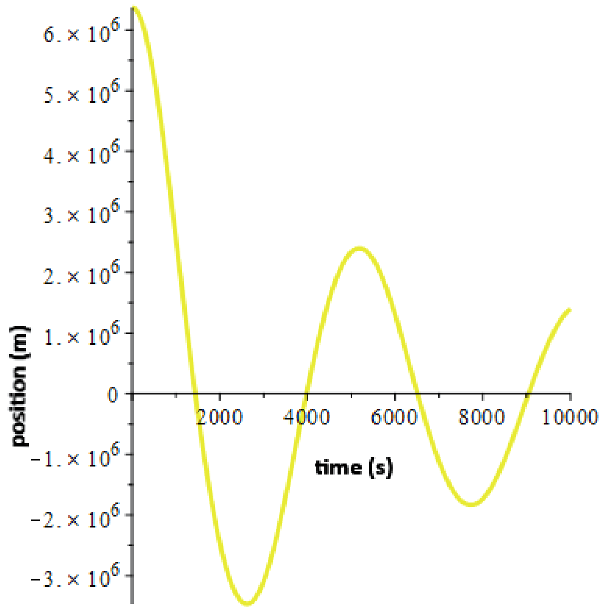

Figure 2: Plot showing the pod position as it travels through the Earth with air resistance.

Conclusion

Building a tunnel through the Earth is to all intents and purposes impossible. However, if it was created, using the Hyperloop concept tube would not be an effective way to travel through the planet. The pod would fall back towards the centre of the Earth after just getting over half way back to the surface. For further research on this topic, the thrust the Hyperloop pod would require to finish its journey could be examined, or a potential investigation into the use of a Hyperloop pod at other planets could be conducted.

References

[1] Charles Q. Choi. How long would it take to fall through the earth?, Mar 2015.

[2] Elon Musk. Hyperloop alpha. *(PDF). SpaceX. Retrieved August*, 13, 2013.